Ontology Languages (COMP321)

Exercise 3

1. Recall the syntax of the Description Logics $\mathcal{EL}$, DL-Lite, and $\mathcal{ALC}$. Suppose Manager and Project are concept names and manages is a role name.

   Let $\mathcal{E}$ be any of the following expressions:

   (a) $\top \sqsubseteq \bot$

   (b) $\exists \mathsf{manages}.\mathsf{Project} \sqsubseteq \mathsf{Manager}$

   (c) $\forall \mathsf{manages}.\mathsf{Project}$

   (d) $\exists \mathsf{Project}.\mathsf{manages}$

   (e) $\exists \mathsf{manages}^{-}.\top \sqsubseteq \mathsf{Project} \sqcup \mathsf{Department}$

   (f) $\mathsf{Manager} \sqsubseteq \exists \mathsf{manages}.\top$

   (g) $\mathsf{Manager} \sqsubseteq \exists \mathsf{manages}.\bot$

   (h) $(\geq\ 7\ \mathsf{manages}.\top) \sqsubseteq \mathsf{Manager}$

   (i) $(\geq\ 8\ \mathsf{manages}.\mathsf{Project}) \sqsubseteq \mathsf{Manager}$

   (j) $\forall \mathsf{manages}.\top \sqsubseteq \exists \mathsf{manages}.\mathsf{Project}$

   (k) $\exists \mathsf{manages}.\top \sqsubseteq (\geq\ 4\ \mathsf{manages}.\top)$.

   (l) $(\geq\ 4\ \mathsf{manages}.\top) \sqsubseteq \exists \mathsf{manages}.\top$.

   - Translate $\mathcal{E}$ into natural language;
   - State whether
     - it is a $\mathcal{EL}$ concept;
     - a $\mathcal{EL}$ concept inclusion;
     - a DL-Lite concept
     - a DL-Lite concept inclusion
     - a $\mathcal{ALC}$ concept
     - a $\mathcal{ALC}$ concept inclusion.
     - none of the above.

- if $\mathcal{E}$ is a concept inclusion, check whether $\mathcal{E}$ follows from the empty TBox (i.e., $\emptyset \models \mathcal{E}$). If this is not the case, define an interpretation $\mathcal{I}$ such that $\mathcal{I} \not\models \mathcal{E}$.
- if $\mathcal{E}$ is a concept, check whether $\mathcal{E}$ is satisfiable. If this is the case, define an interpretation $\mathcal{I}$ such that $\mathcal{E}^{\mathcal{I}} \neq \emptyset$.

2. Show that every $\mathcal{EL}$-TBox is satisfiable. In other words, show that for every $\mathcal{EL}$-TBox $\mathcal{T}$ there exists an interpretation $\mathcal{I}$ such that $\mathcal{I} \models \mathcal{T}$.