Distributed Computation and Reconfiguration in Actively Dynamic Networks

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Dynamic Networks

General Description:

- Networks whose structure may change
- Usually represented by a graph
- Edges and/or nodes come and go

Dynamics:

- Active or Passive
- Centralized or Distributed

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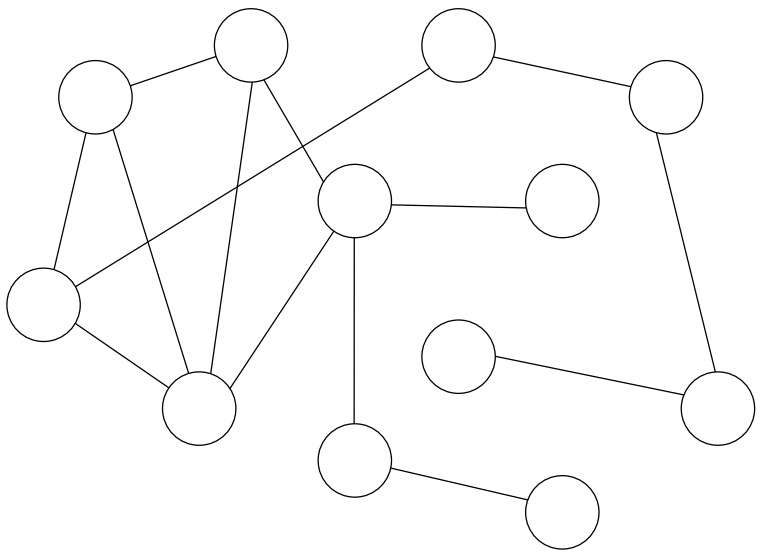

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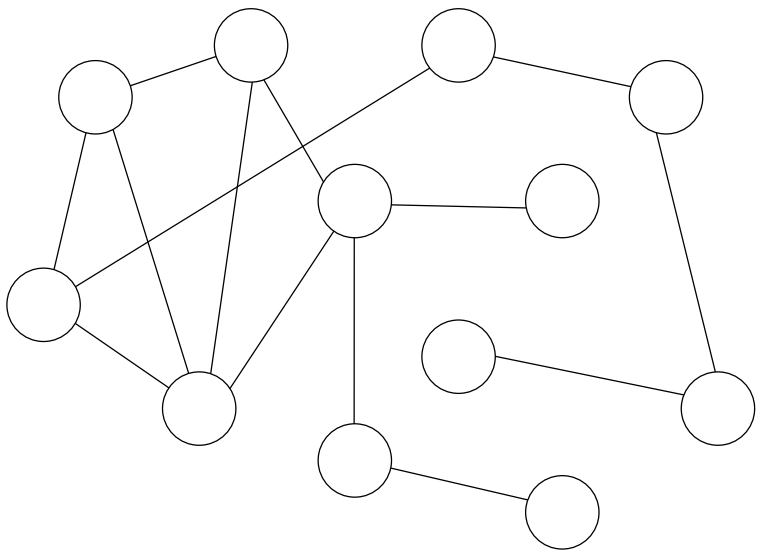

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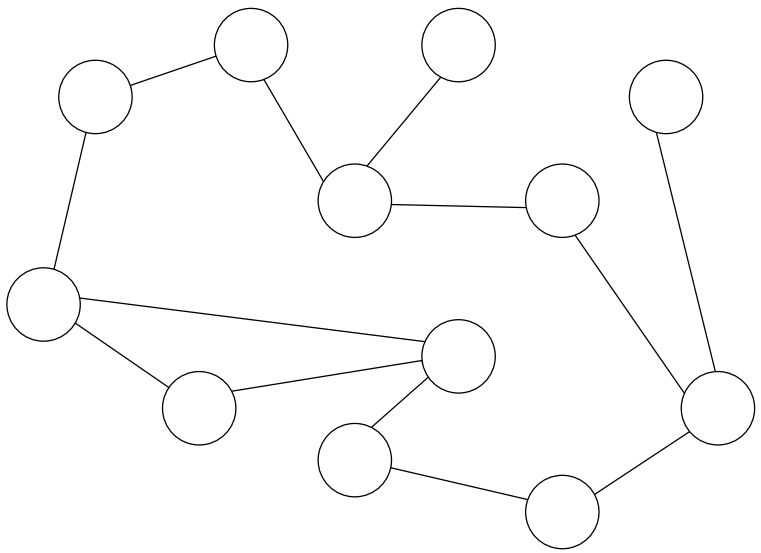

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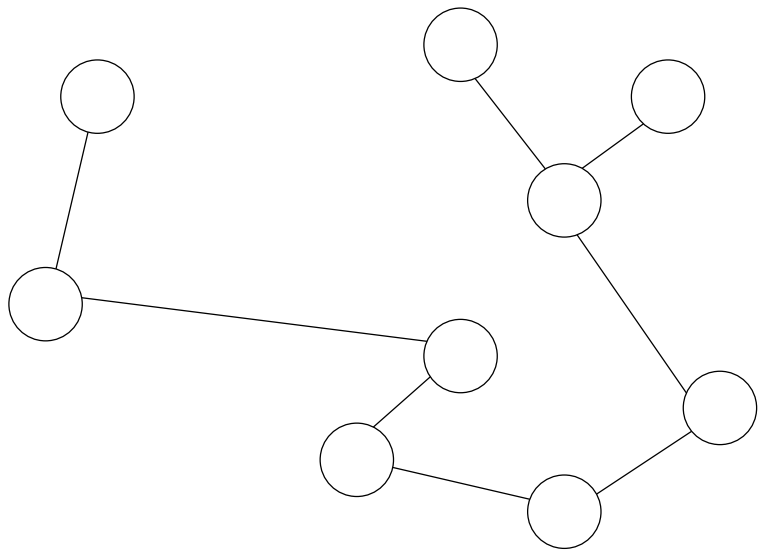

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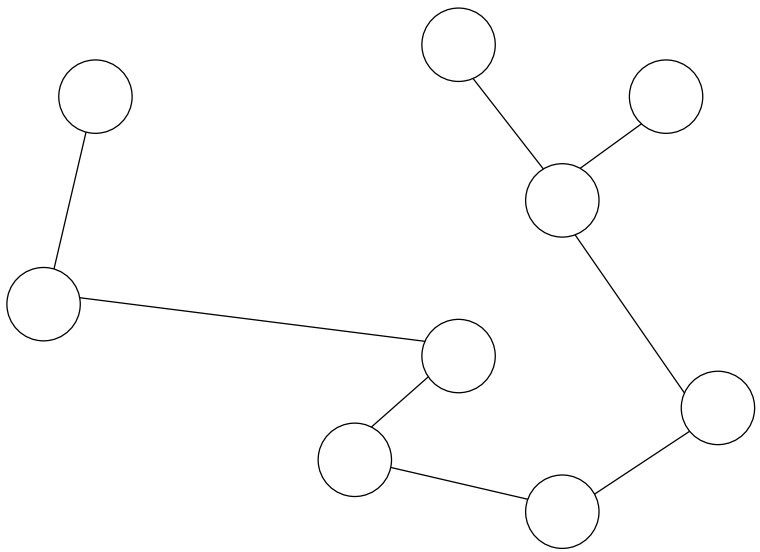

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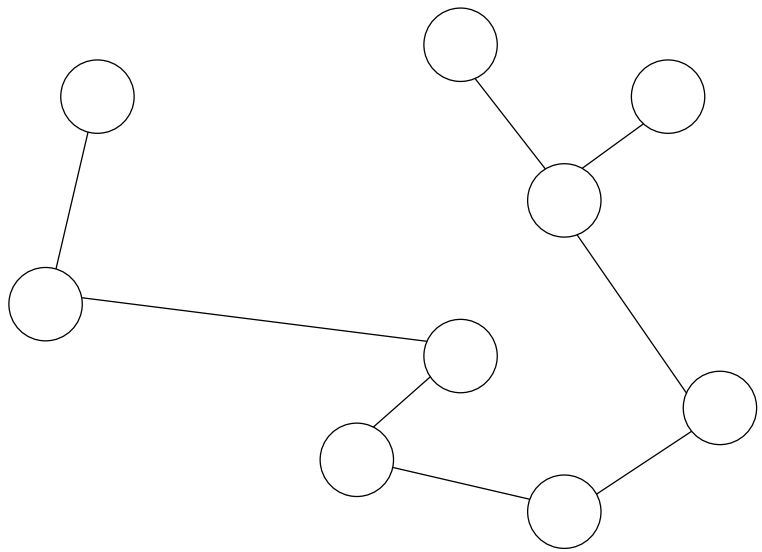

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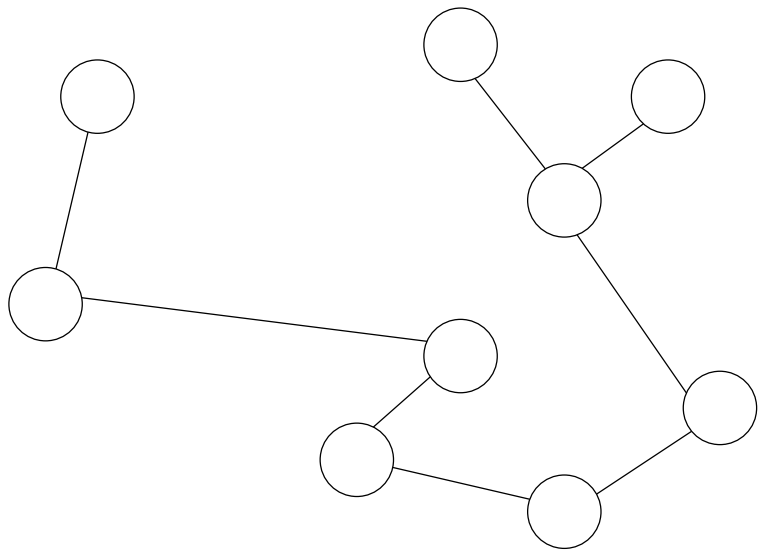

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Active Dynamics with Distributed Control

- Activate connections to new neighbors or deactivate connections
- No central coordinator

General Goal:

- Transform the initial network G_s into a target network G_f from a family of target networks
- Exploit some good properties of G_f in order to more efficiently solve a distributed task

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Related Systems

- Temporal Graphs
  - Initiated by Berman [Be09] and Kempe *et al.* [KKK00]
- Distributed Computation in Passively Dynamic Networks
  - Kuhn *et al.* [KLO10]
  - Population Protocols [AADFP06]
- Construction of Overlay Networks
  - Most similar to our model
  - Introduced by Stoica *et al.* [SMK01] and [AS07]
- Programmable Matter
  - Geometry plays a big role

Our Model

- Initial connected network G_s
- Static set V of n nodes
- Dynamic set $E(i)$ of m active edges
- Standard synchronous message passing model
- Each node has a unique ID
- Nodes can only compare unique IDs
- Node u can activate an edge with node v if they have a common neighbor
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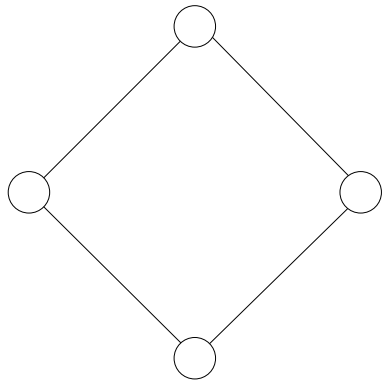

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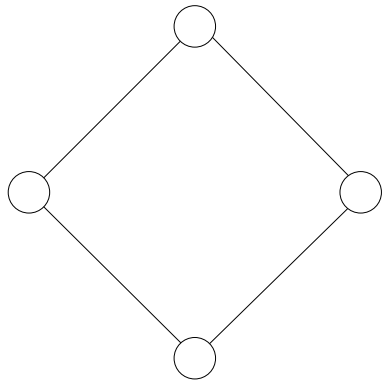

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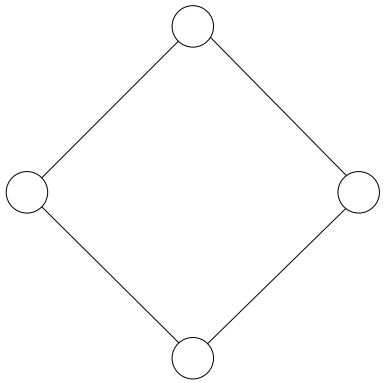

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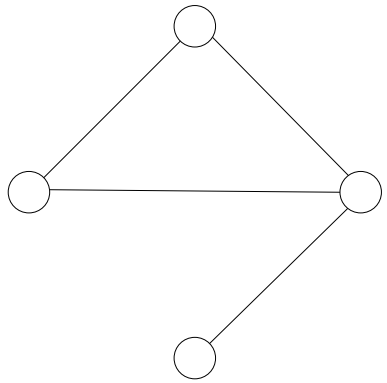

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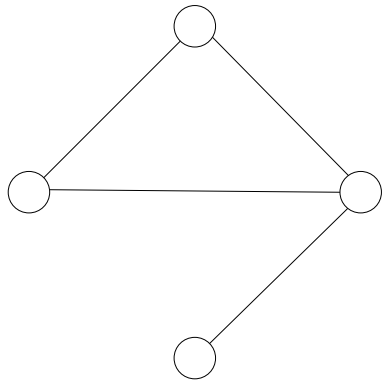

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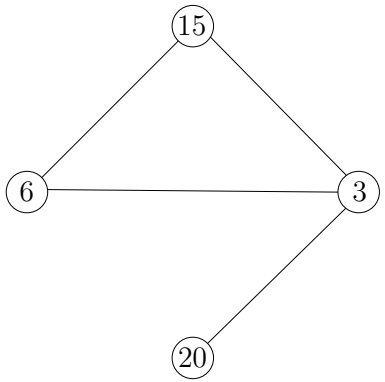

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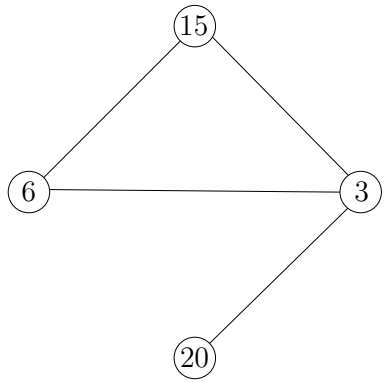

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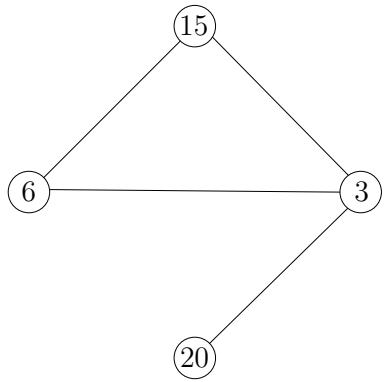

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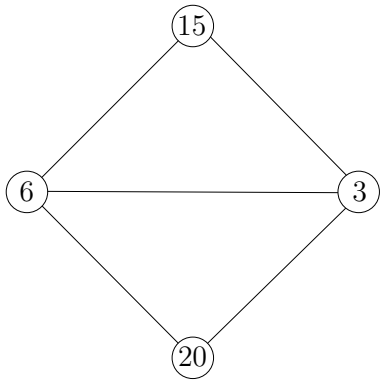

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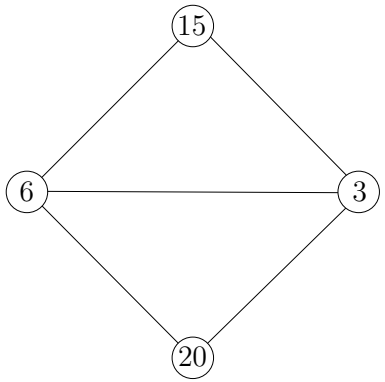

The Problems

Problems:

- Leader Election: Elect a unique leader
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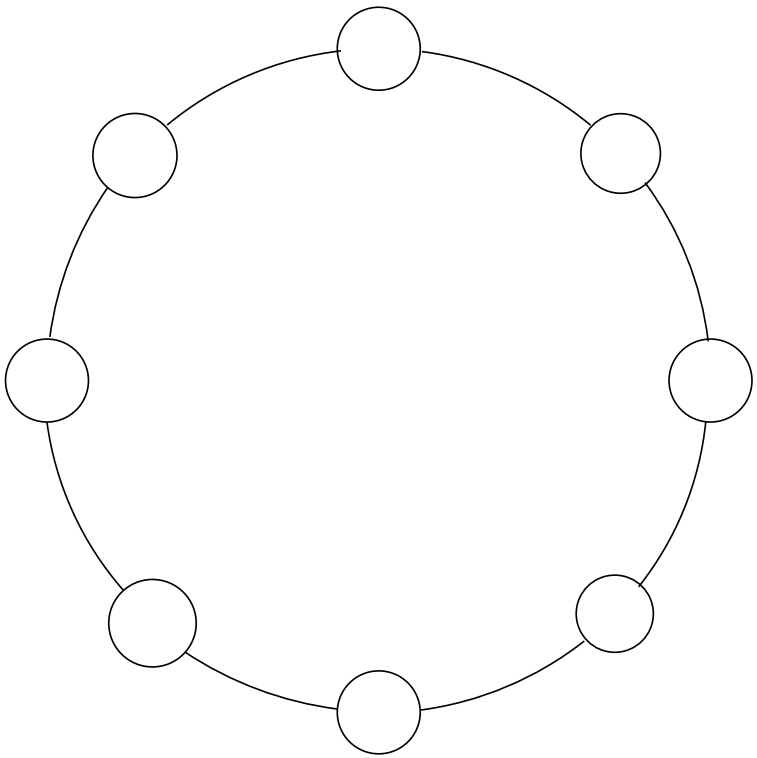

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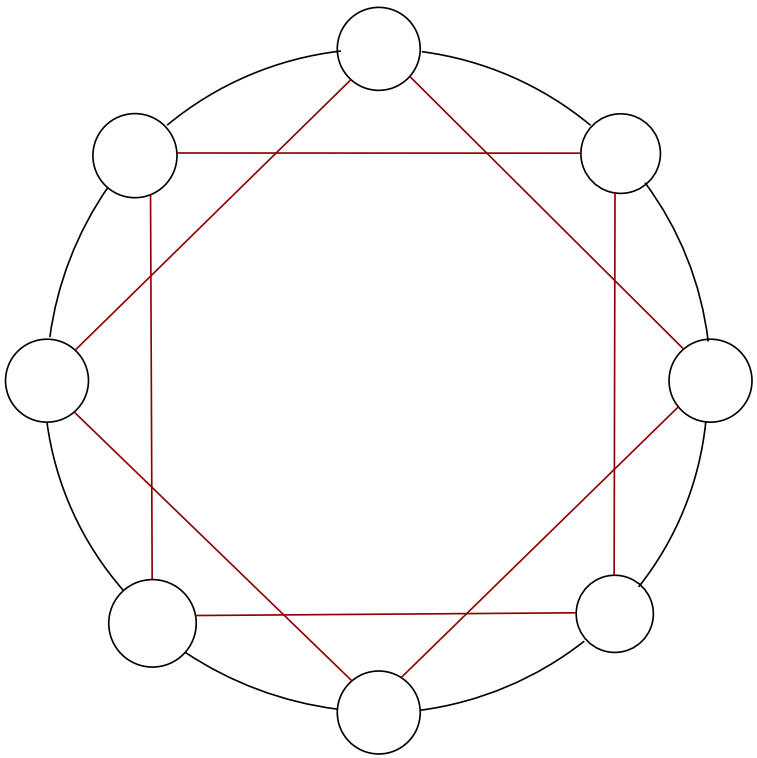

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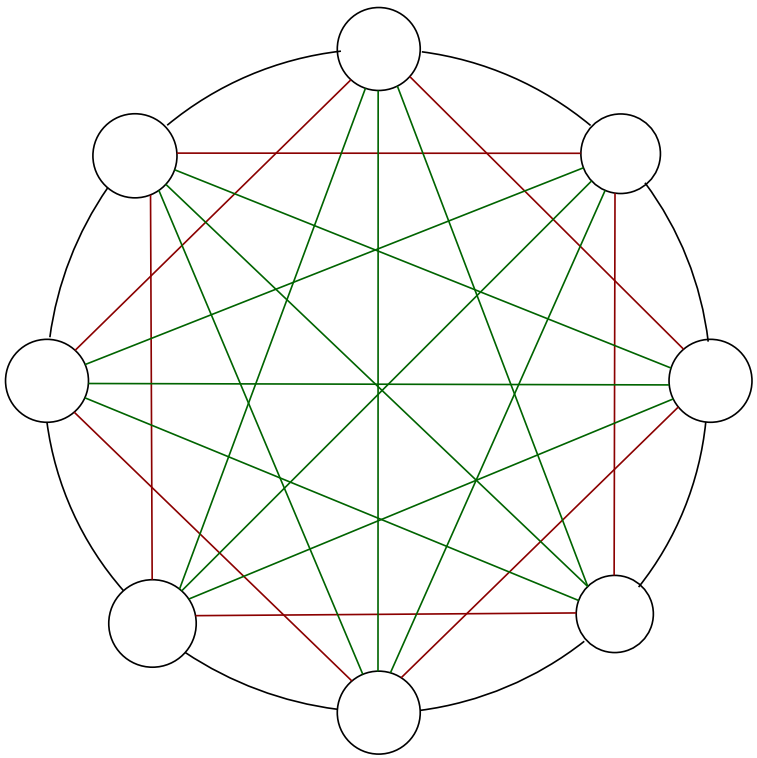

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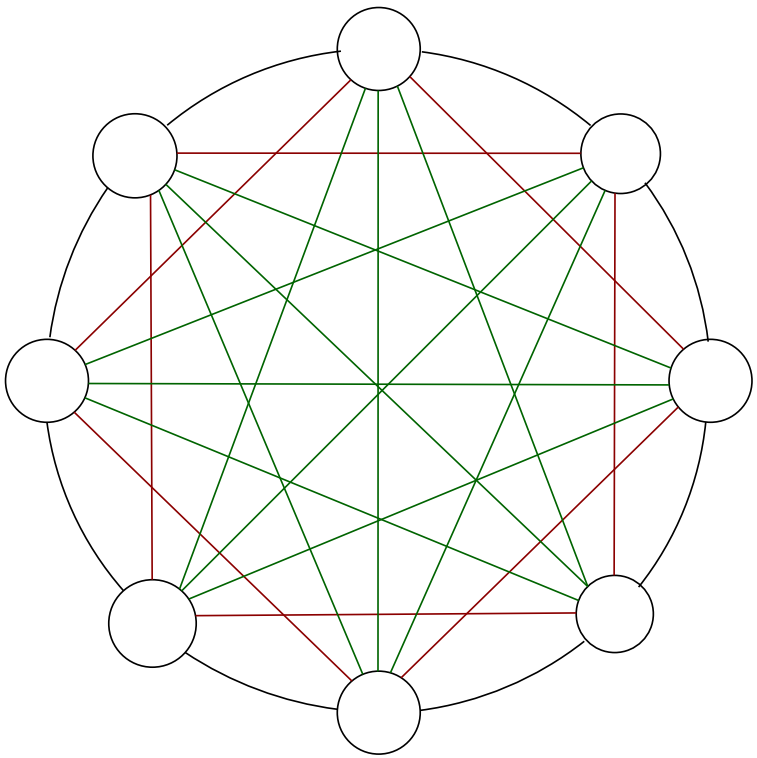

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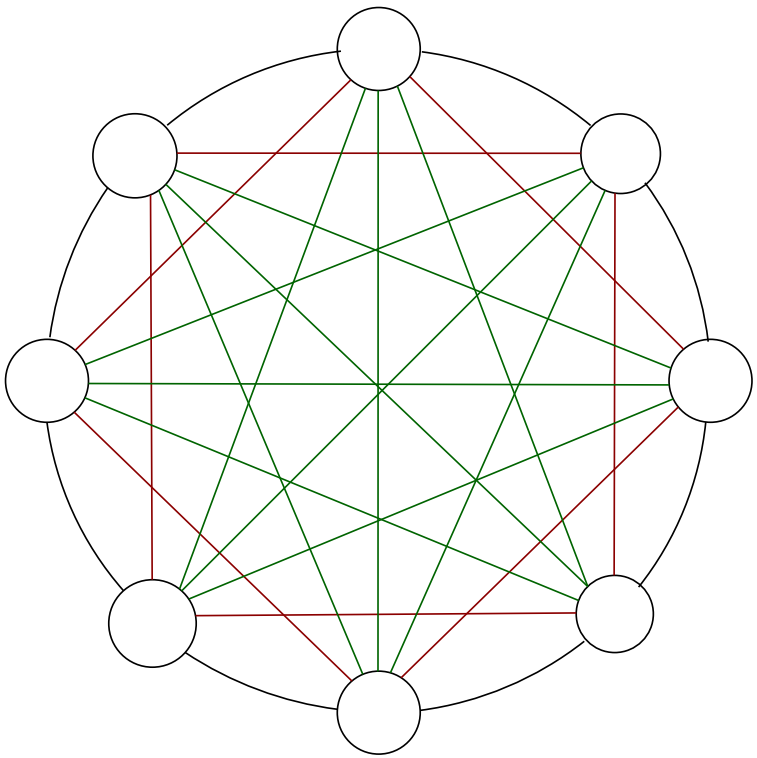

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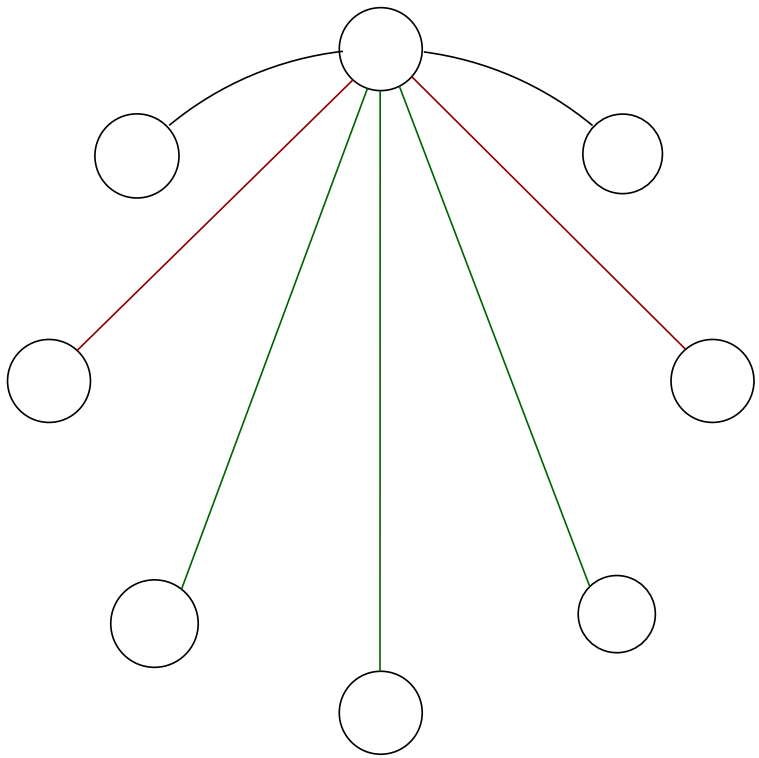

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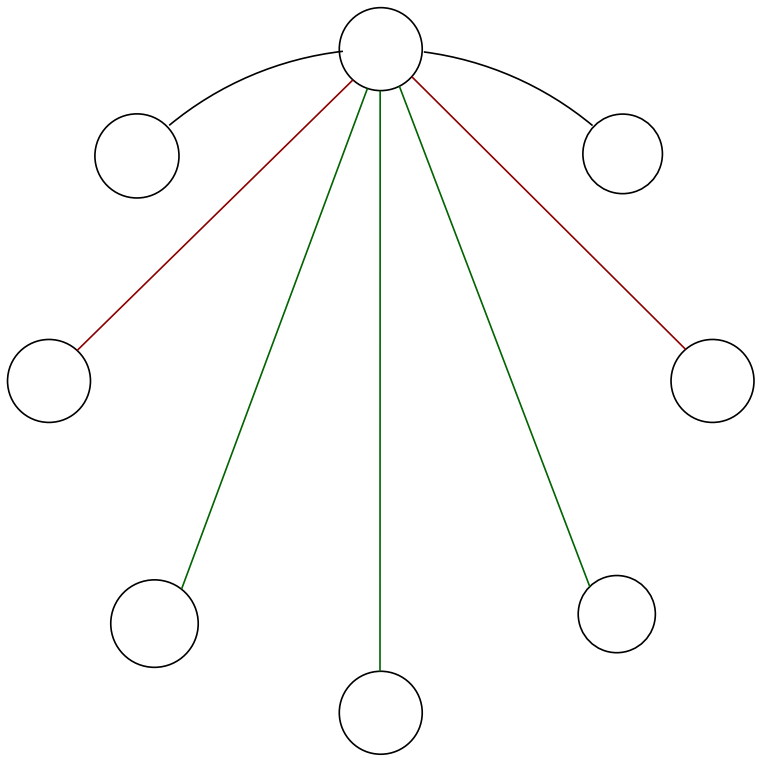

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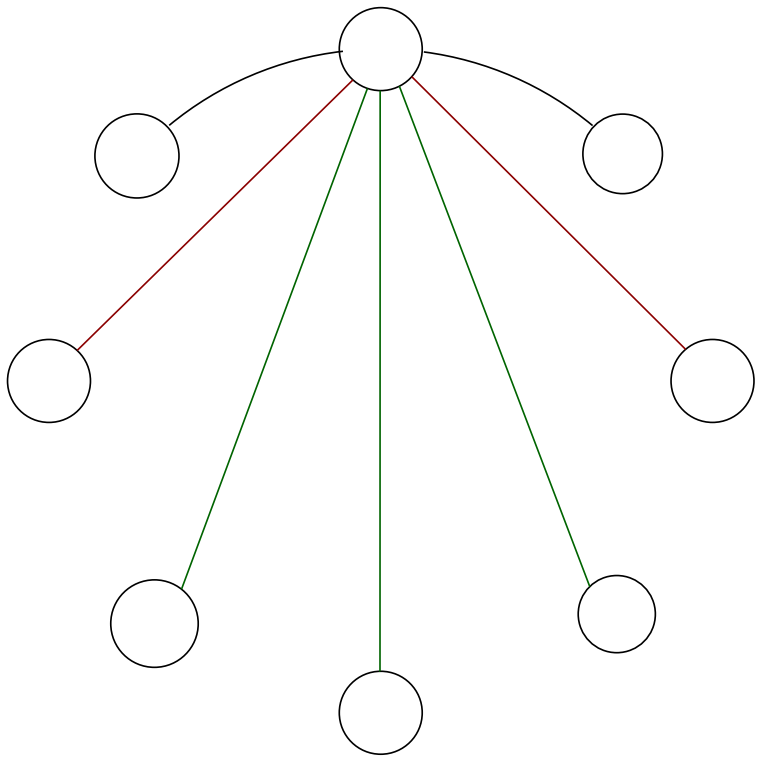
- Activating and maintaining a connection does not come for free



The Measures

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- Total Edge Activations: The number of edges activated by the algorithm
- Maximum Activated Edges: The maximum number of activated edges by the algorithm per round
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Our Results

Algorithm	Time	Total Edge Activations	Maximum Activated Edges	Maximum Activated Degree
GraphToStar	$O(\log n)$	$O(n \log n)$	$O(n)$	$O(n)$
GraphToWreath	$O(\log^2 n)$	$O(n \log^2 n)$	$O(n)$	$O(1)$
GraphToThinWreath	$O(\log^2 n / \log \log n)$	$O(n \log^2 n)$	$O(n)$	$O(\log^2 n)$

Table: Algorithms

Bounds	Time	Total Edge Activations	Maximum Activated Edges	Maximum Activated Degree
Centralized Lower	$\Omega(\log n)$	$\Omega(n)$	$\Omega(n / \log n)$	
Centralized Upper	$O(\log n)$	$\Theta(n)$		
Distributed Lower	$O(\log n)$	$\Omega(n \log n)$		

Table: Bounds for Depth-d tree

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- Nodes are partitioned into committees
- Committees organized into gadget networks
- Each node forms its own committee
- Committees compete and merge
- Final network has a single committee
- Logarithmic running time
- Time: phases*gadgetdiameter

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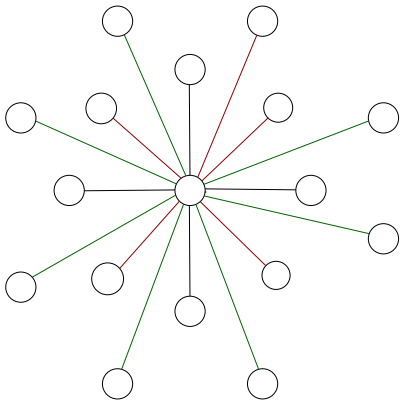

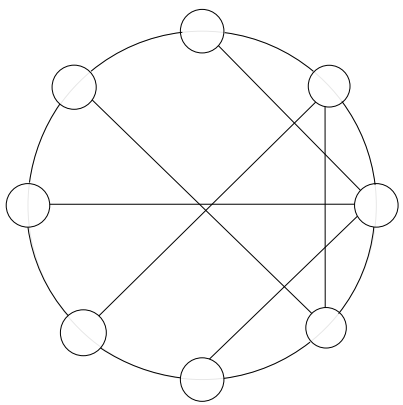

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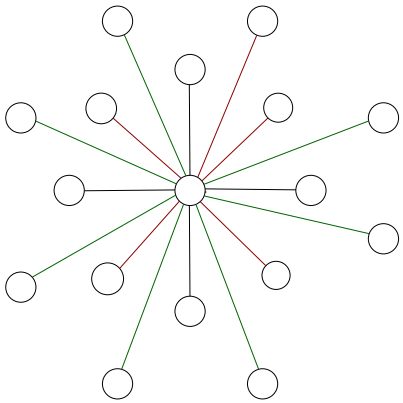

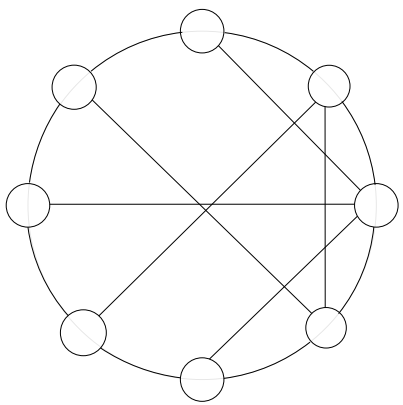

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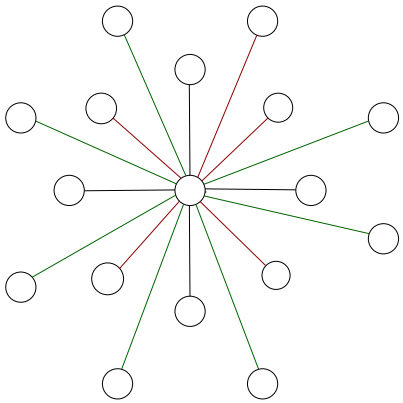

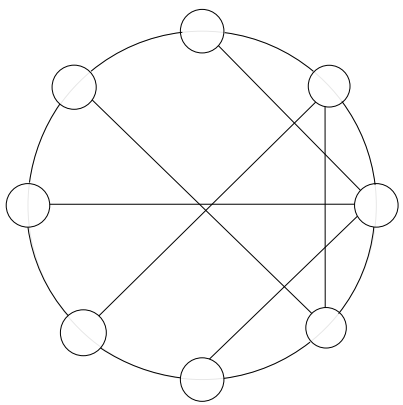

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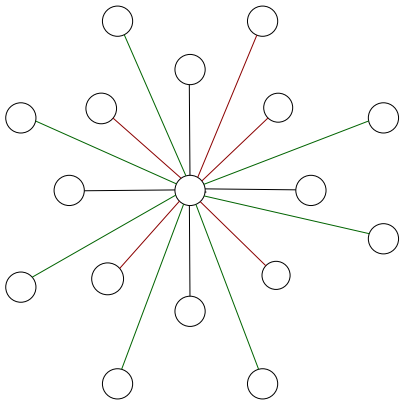
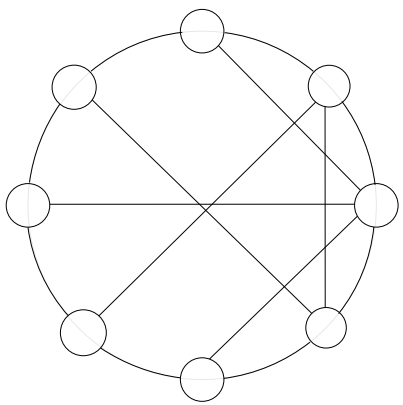
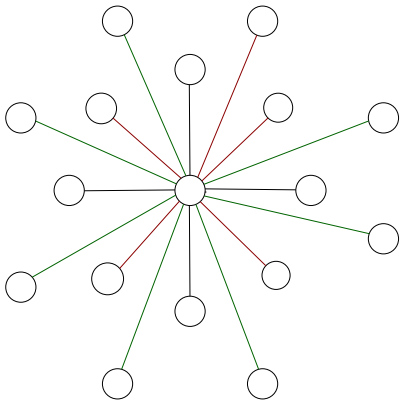
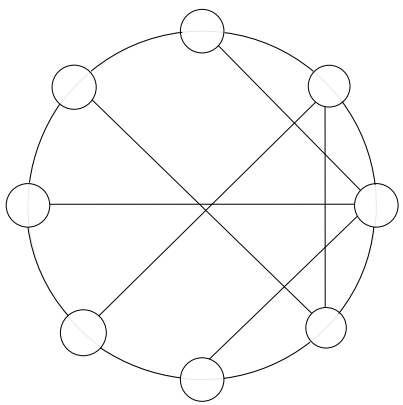




Introduction on the General Strategy

- Nodes are partitioned into committees
- Committees organized into gadget networks
- Each node forms its own committee
- Committees compete and merge
- Final network has a single committee
- Logarithmic running time
- Time: phases*gadgetdiameter





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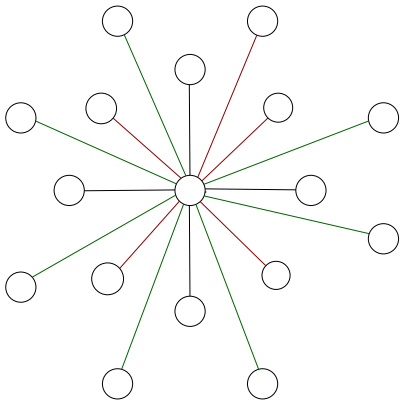

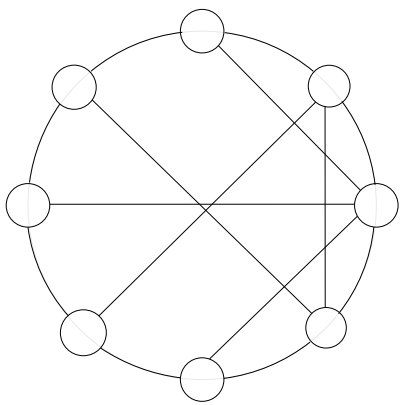

GraphToStar Algorithm

- Gadget network: Star
- TreeToStar subroutine

TreeToStar

- Transforms any initial oriented Tree graph into a spanning Star graph in $\log n$ time
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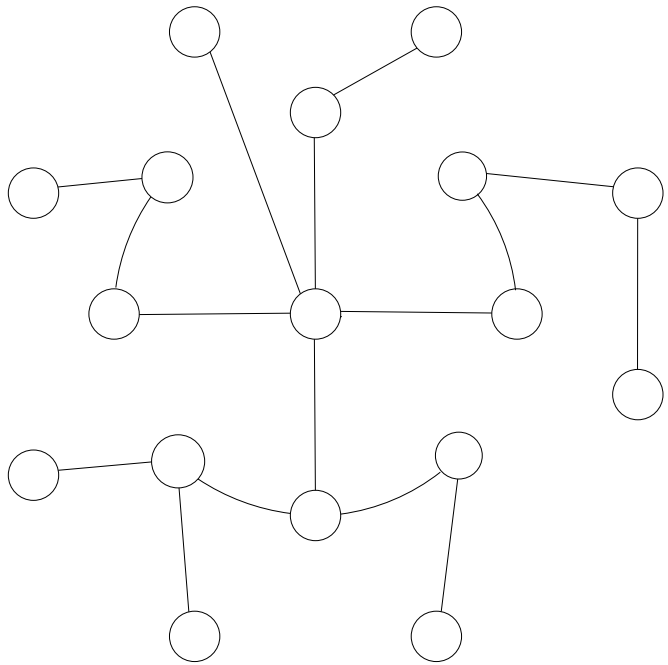

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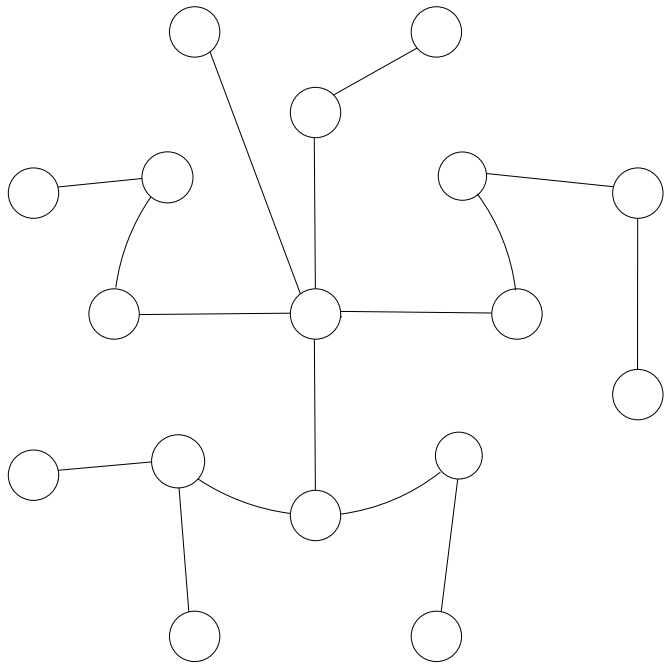

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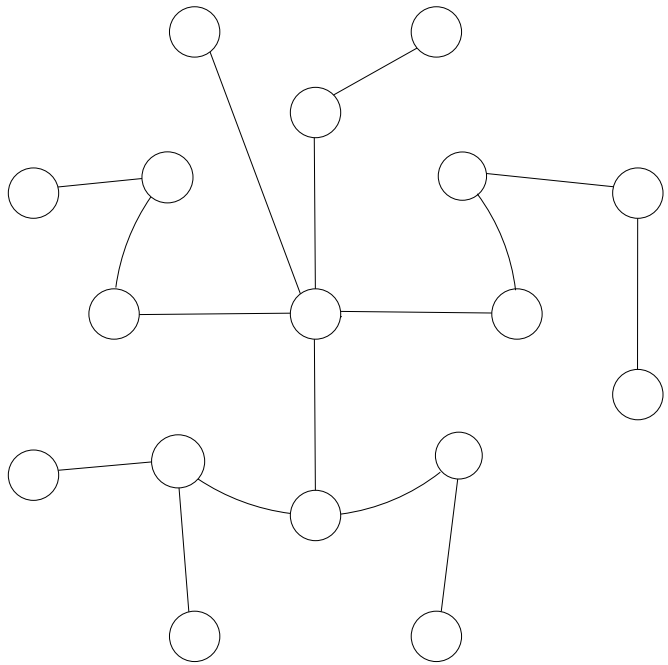

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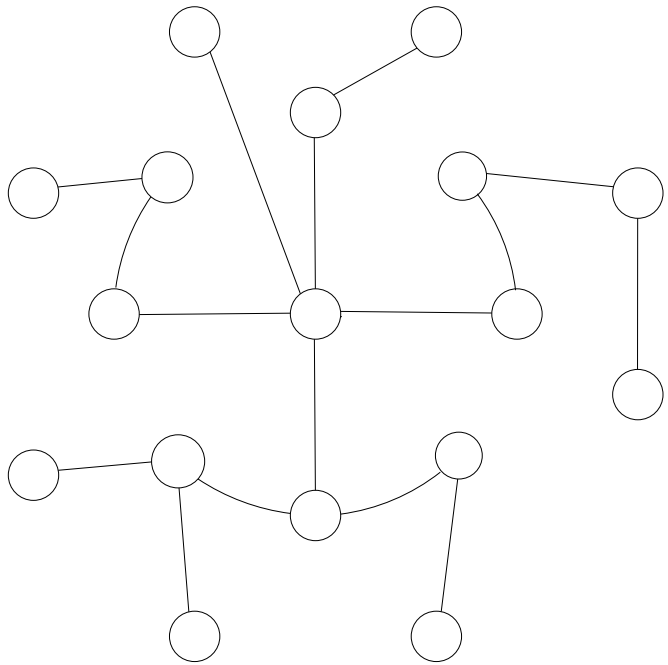

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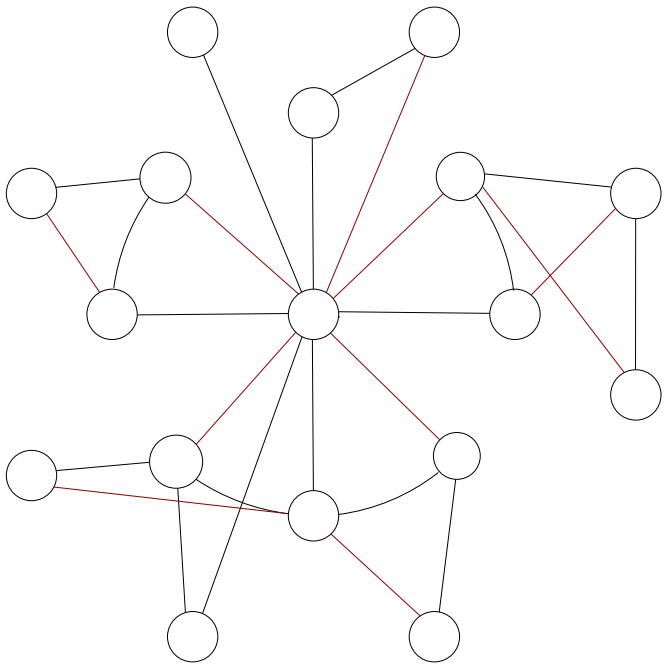

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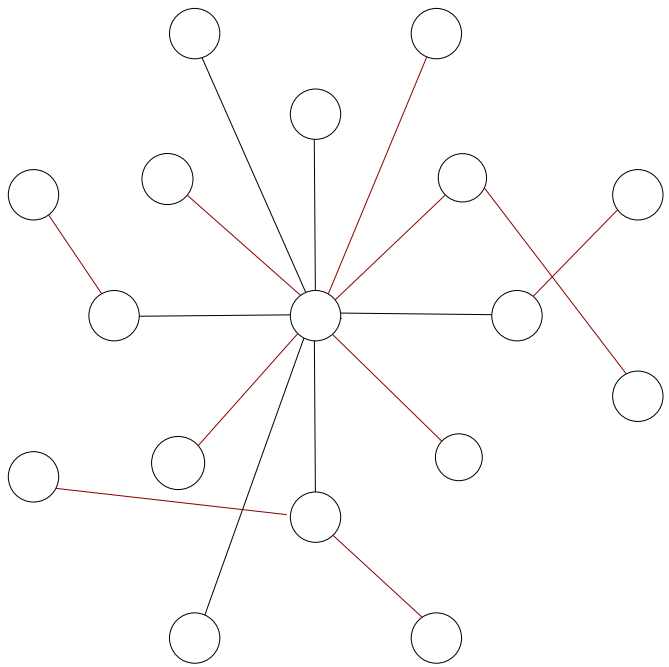

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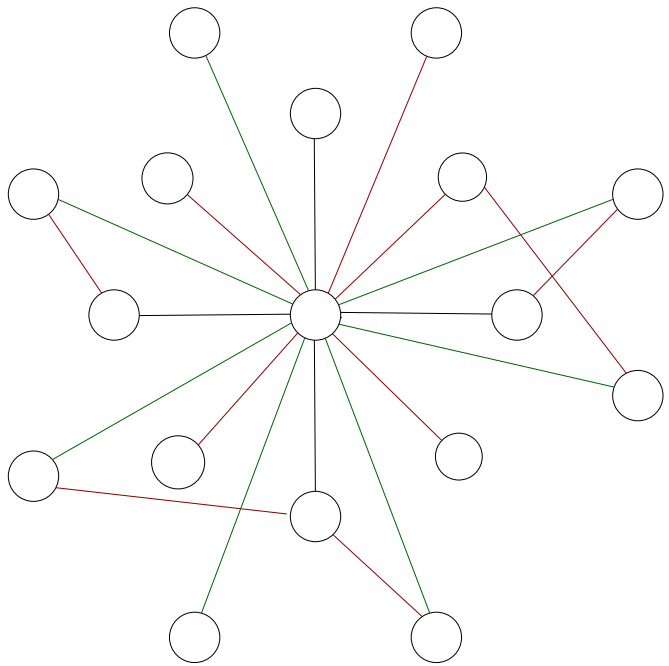

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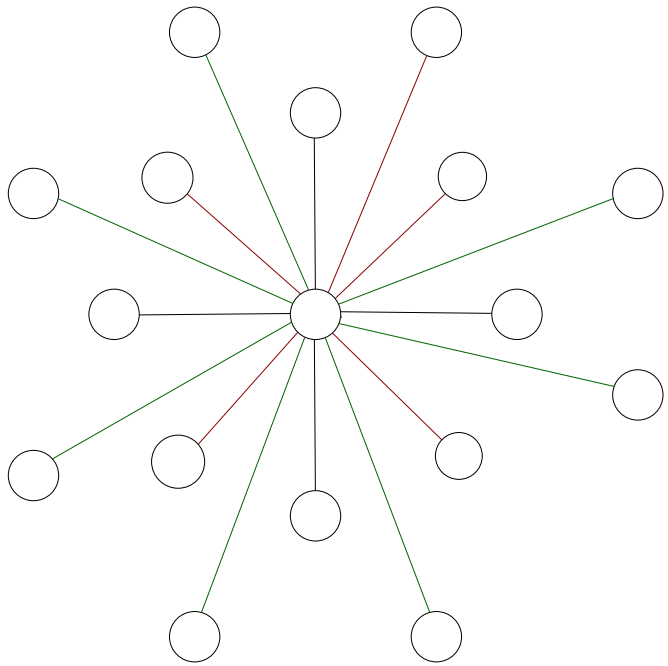

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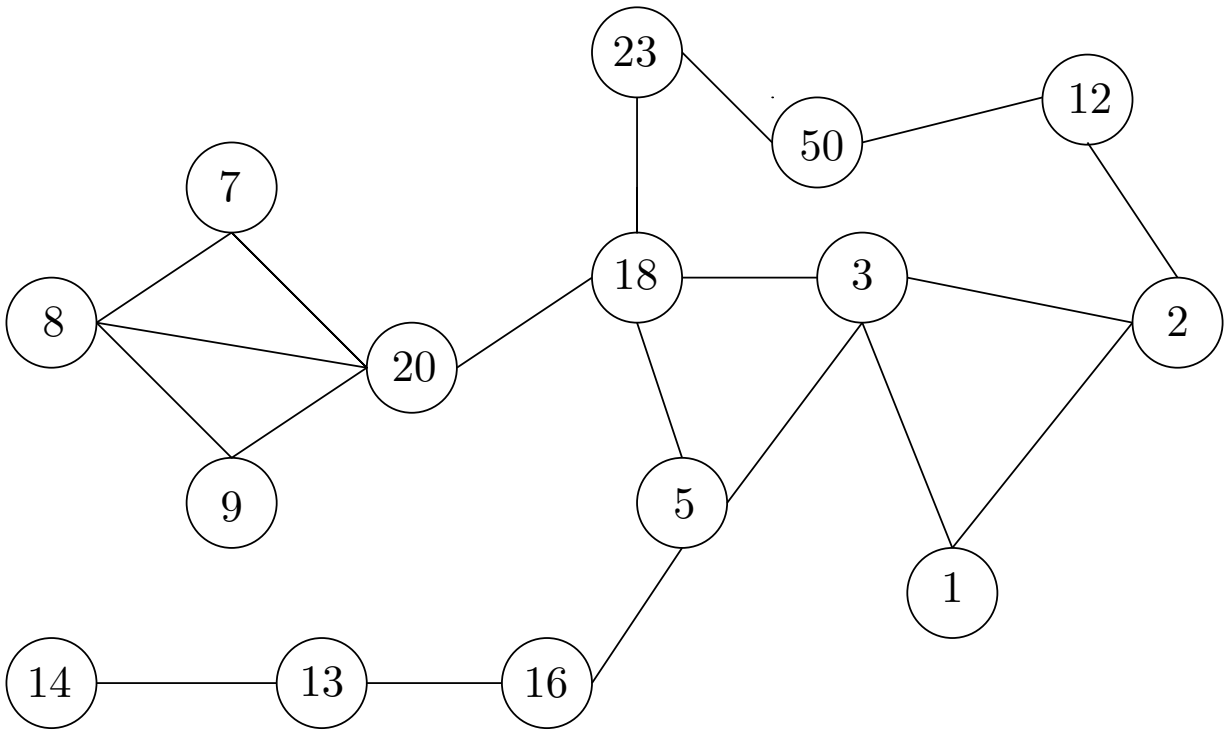

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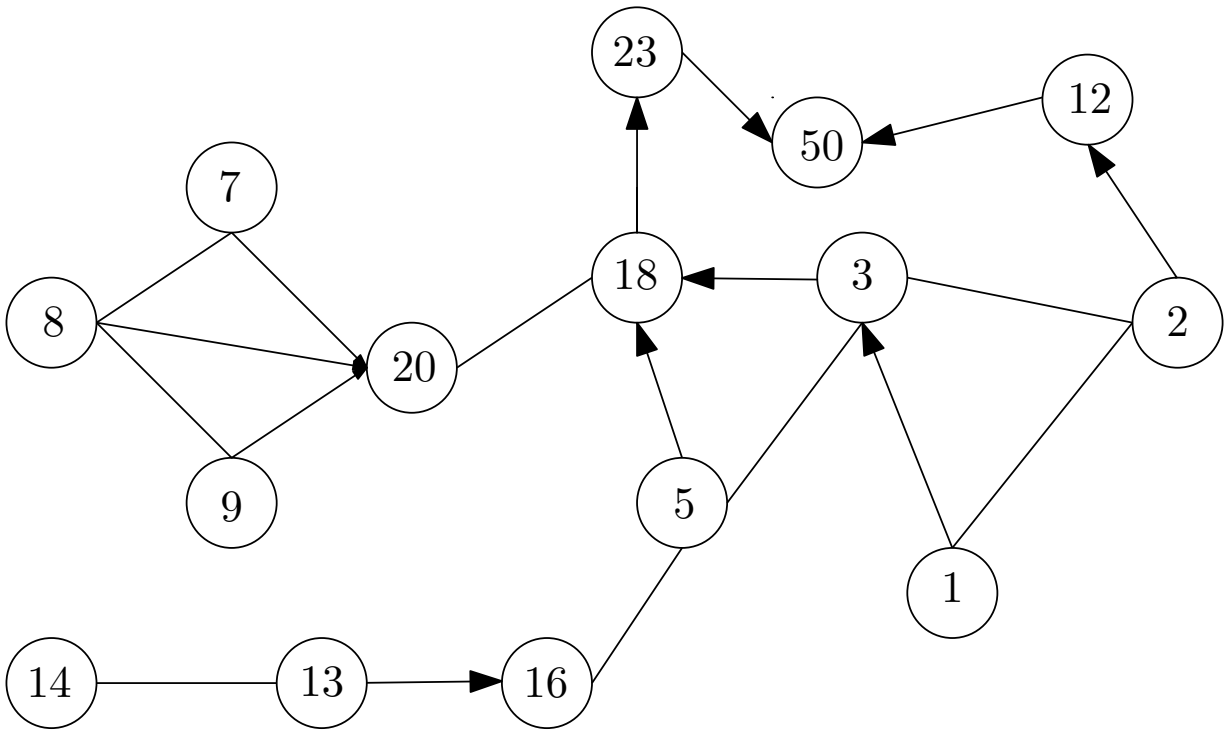

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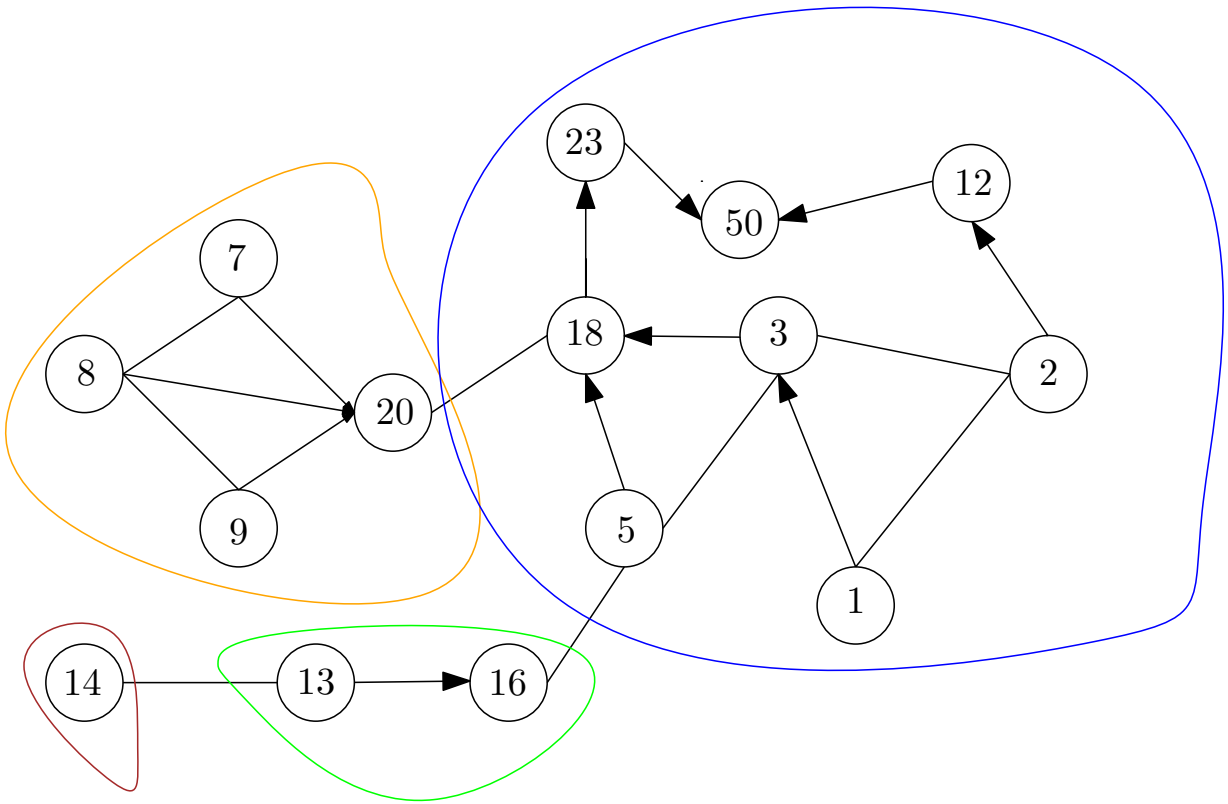

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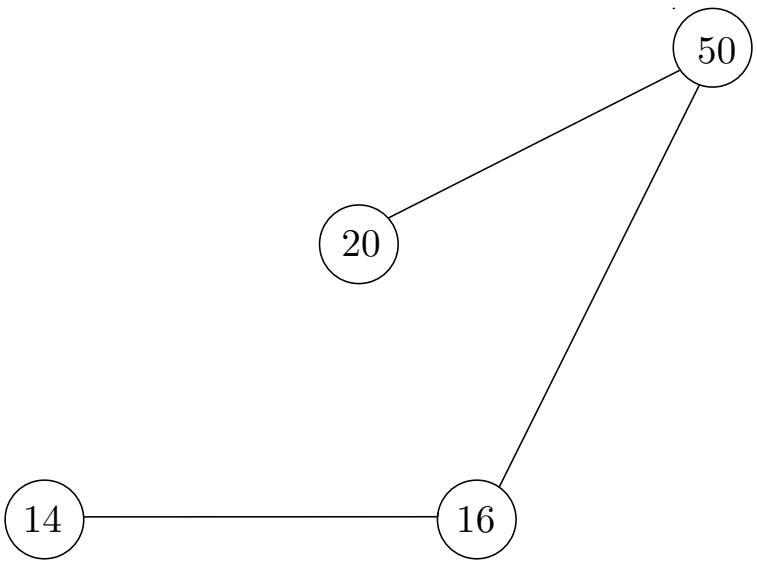

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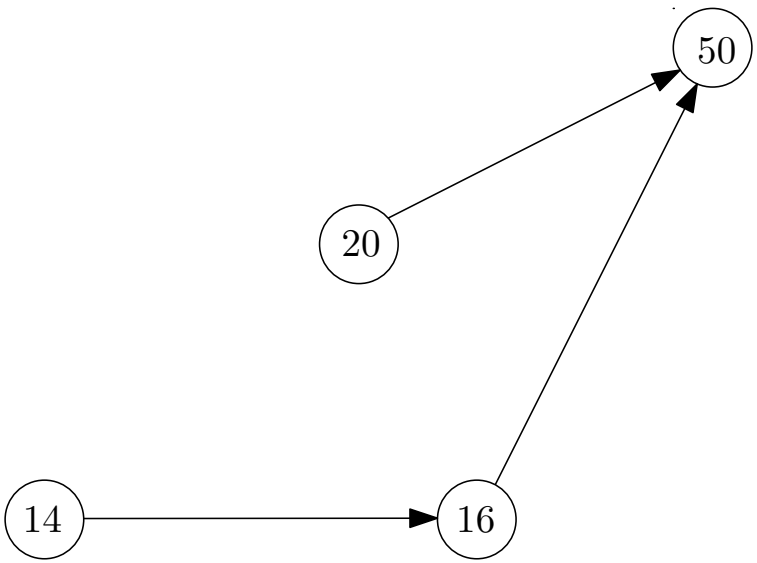

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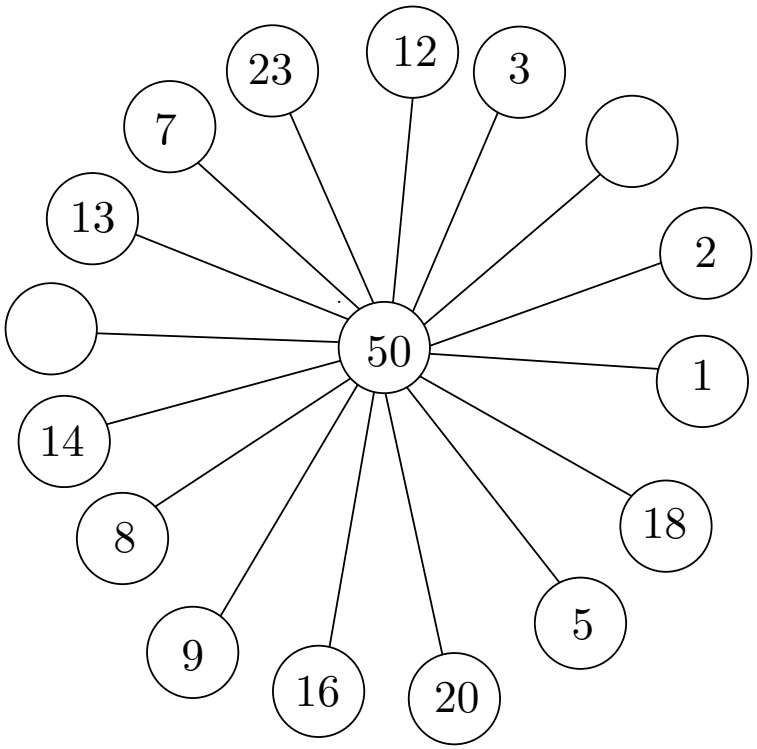


GraphToStar Proof Sketch

Theorem: For any initial connected graph G_s, the GraphToStar algorithm solves the Depth-1 Tree problem in $O(\log n)$ time with at most $O(n \log n)$ total edge activations and $O(n)$ active edges per round

Correctness

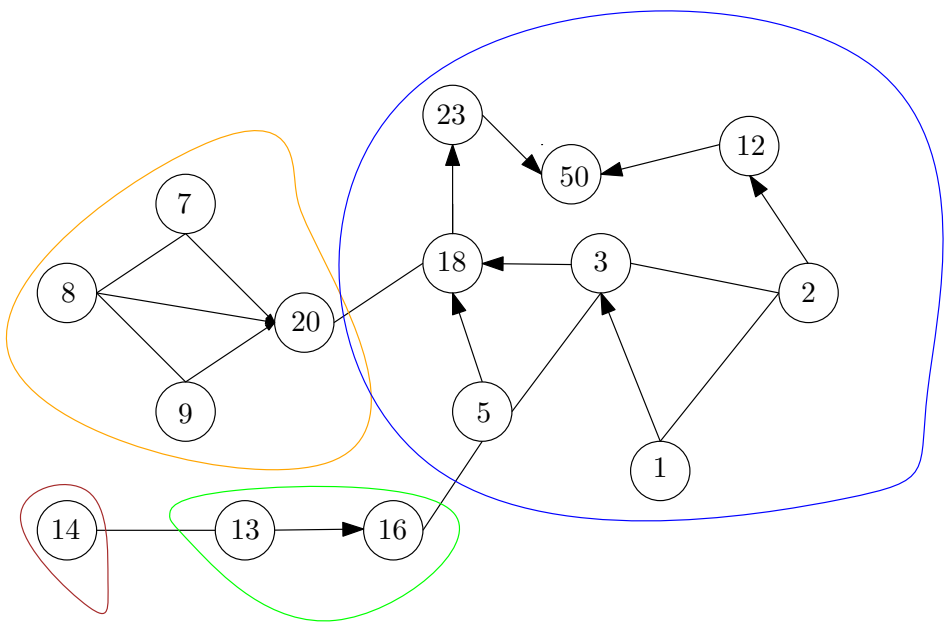
- Committees keep merging
- Can't get isolated indefinitely

Time Complexity

- Committees "double" in size
- Isolated

Edge Complexity

- Omitted



GraphToWreath Algorithm

- Gadget network: Wreath
- LineToCompleteBinaryTree subroutine

LineToCompleteBinaryTree

- Transforms any initial oriented line into a spanning Complete Binary Tree in $\log n$ time
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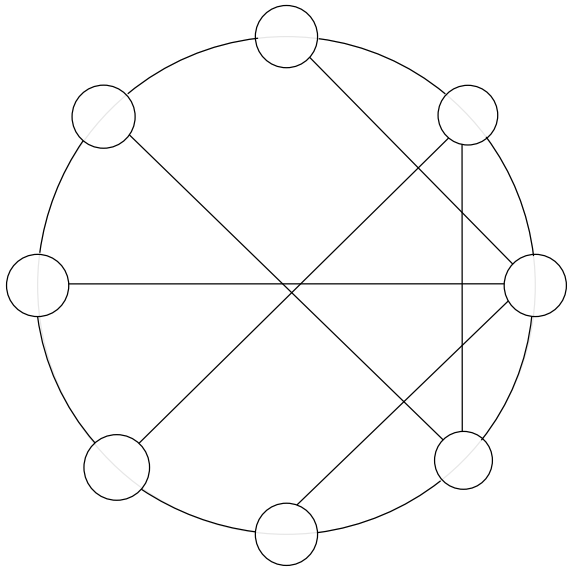

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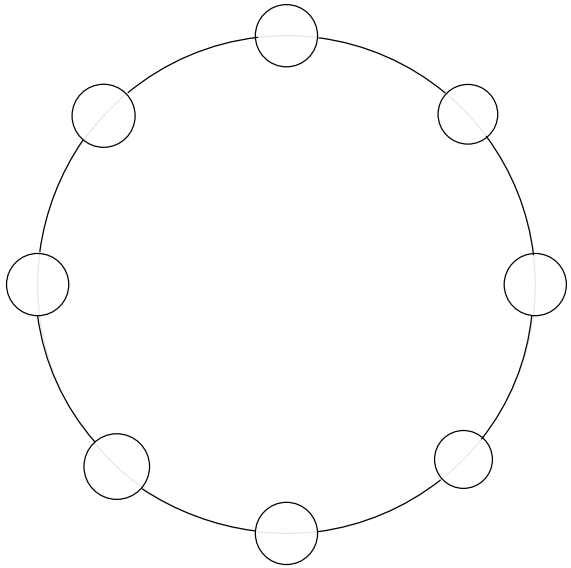

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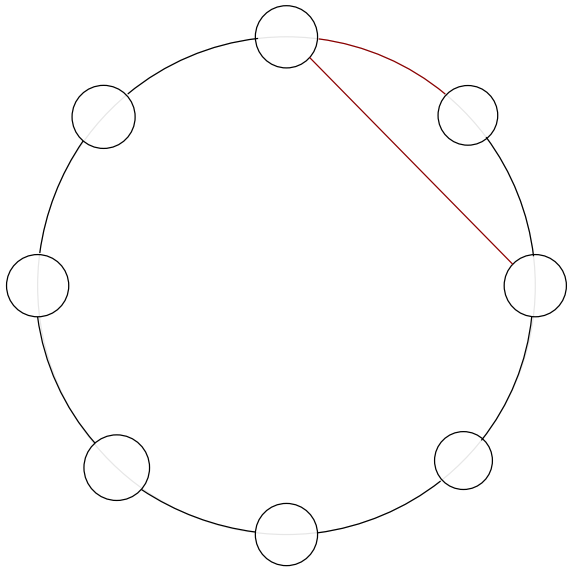

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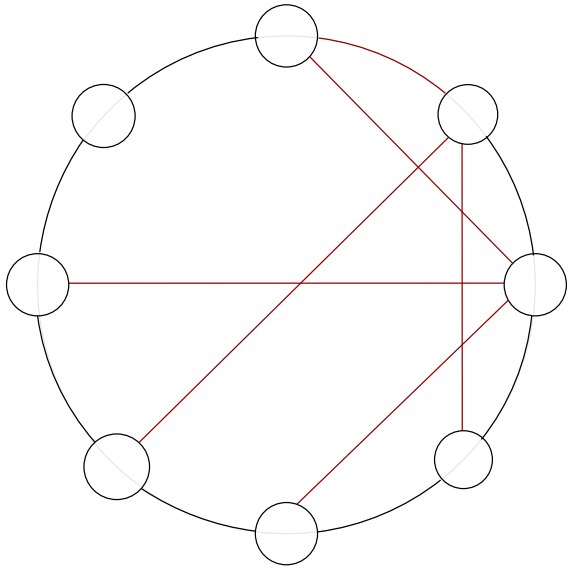

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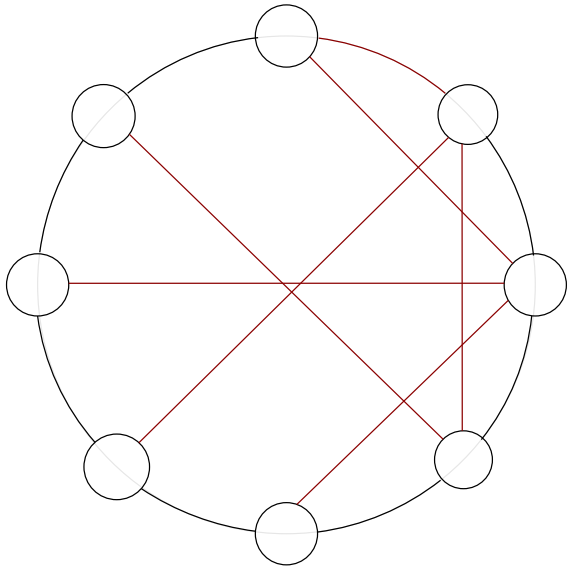

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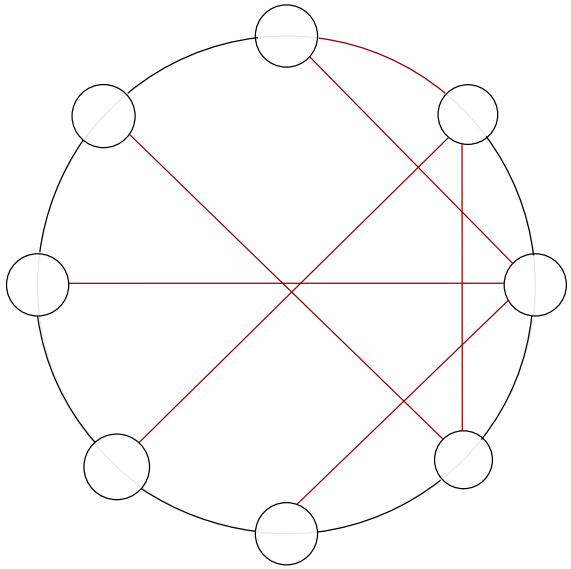

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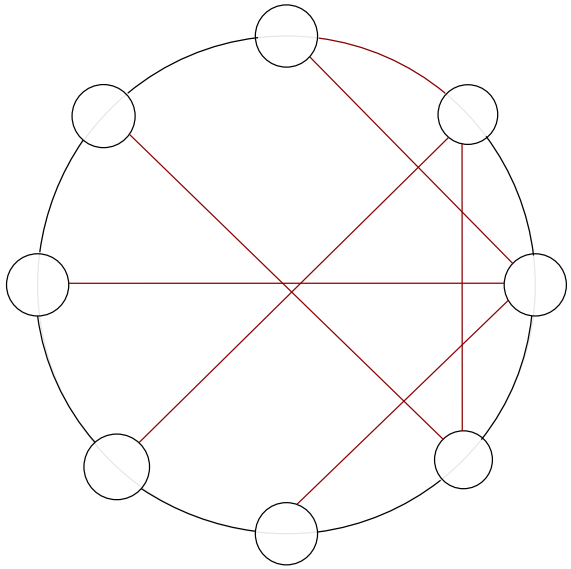

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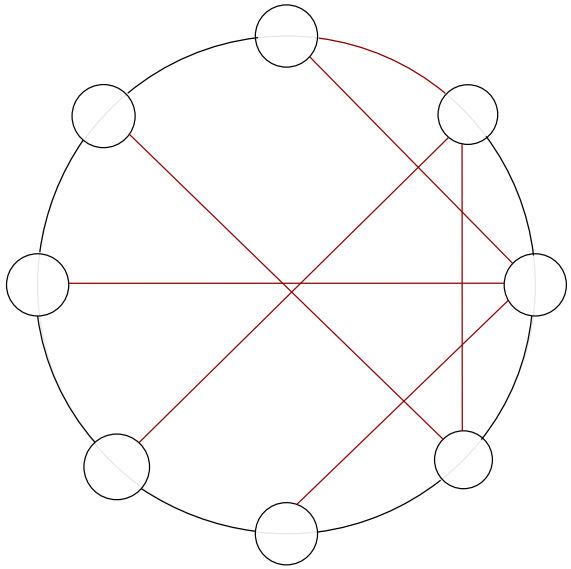

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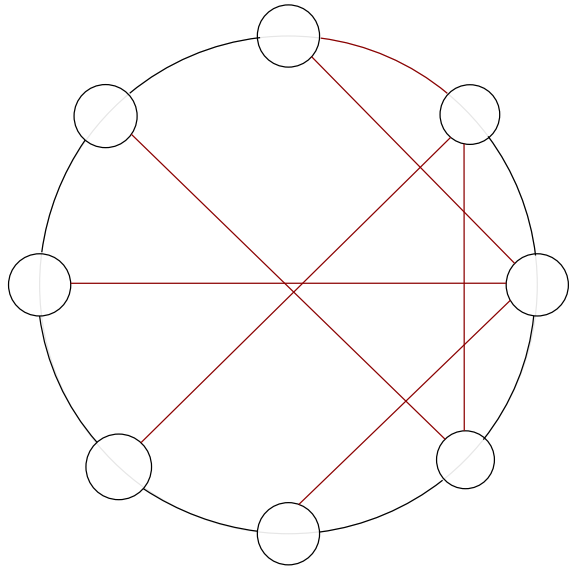

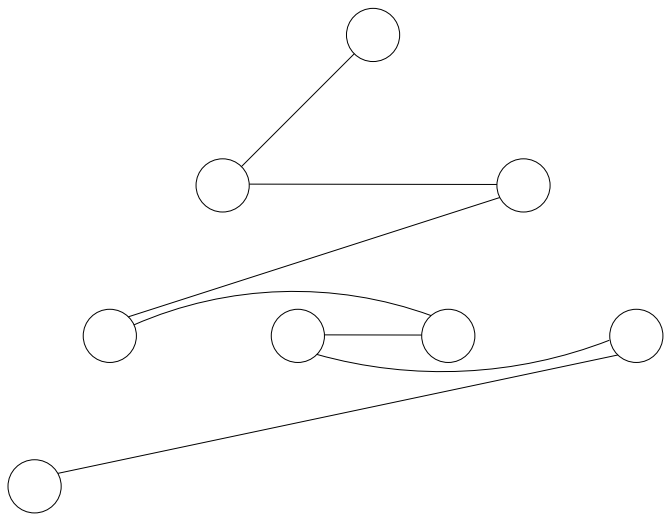

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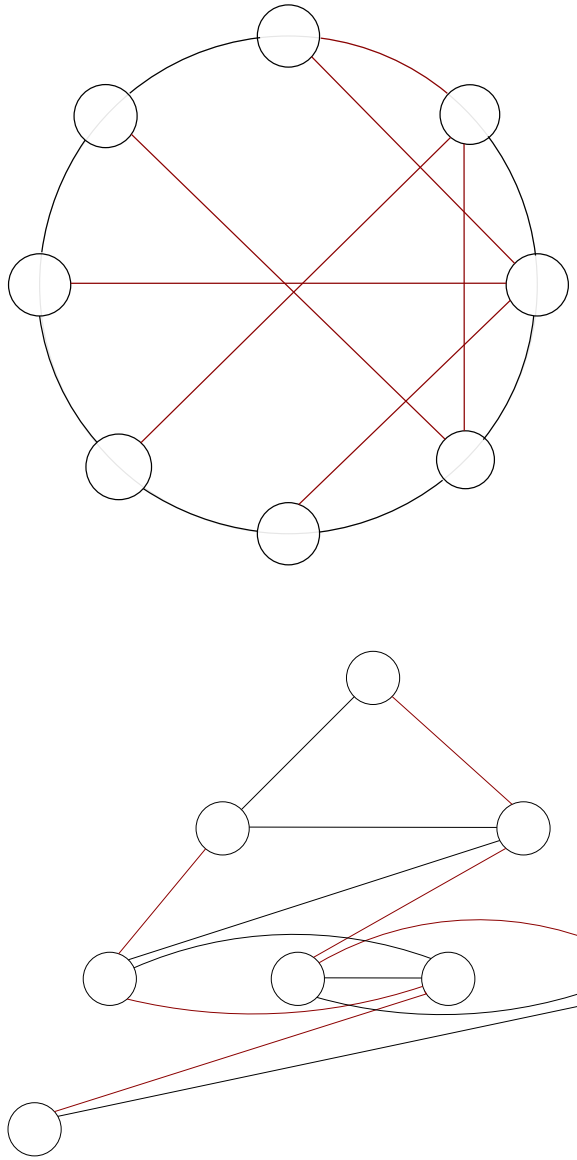

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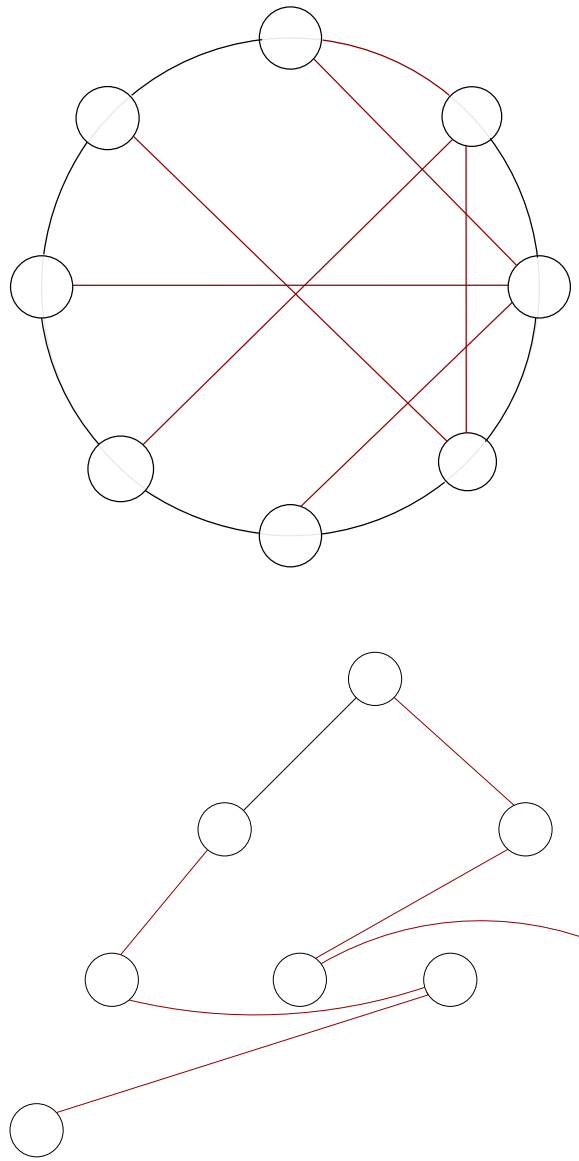

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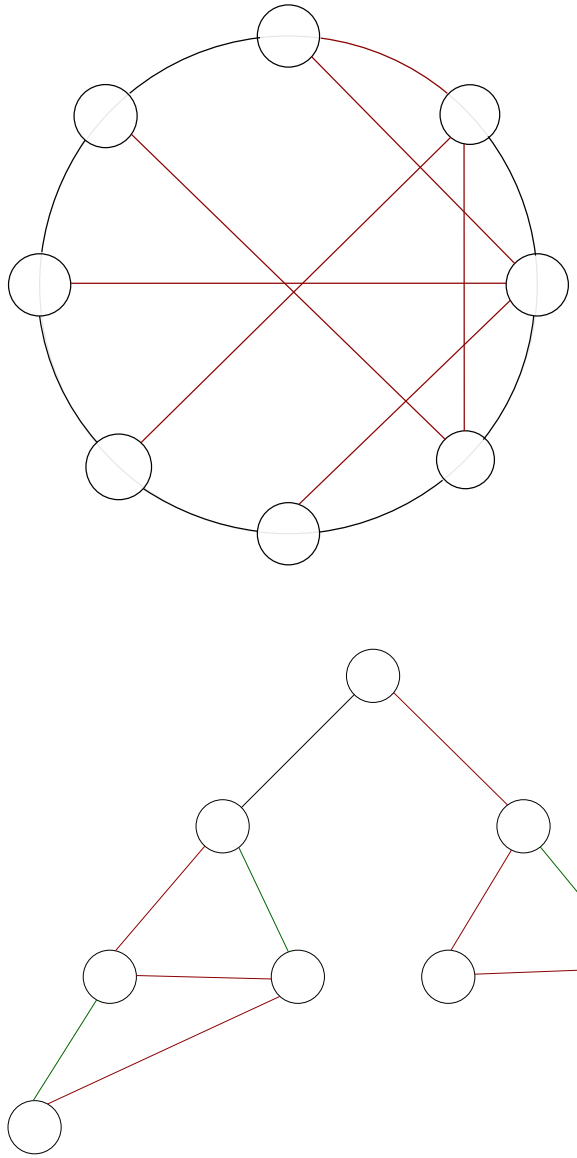

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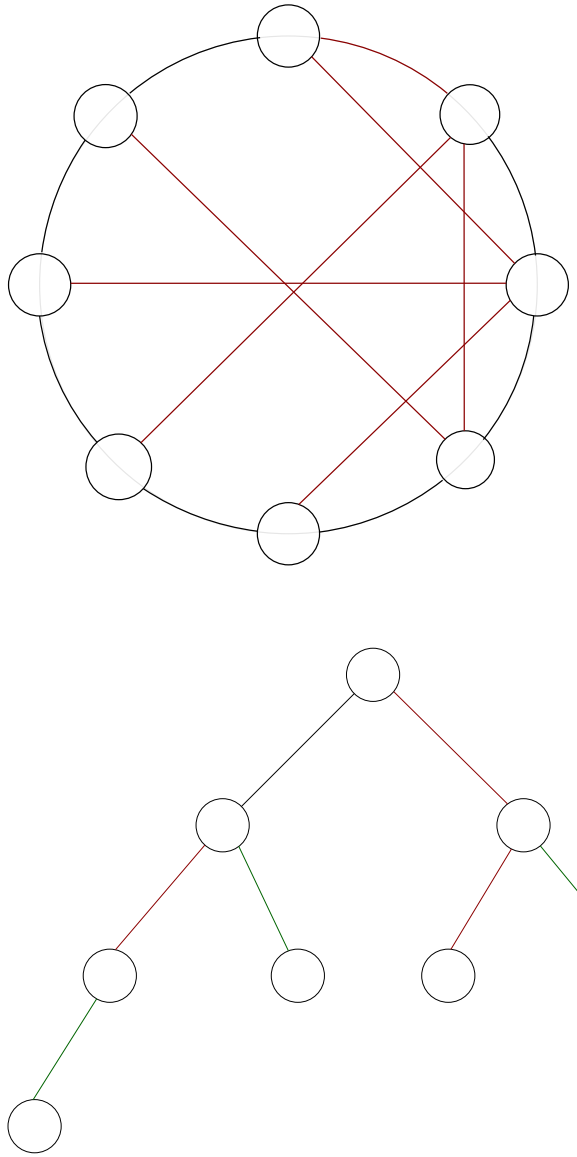

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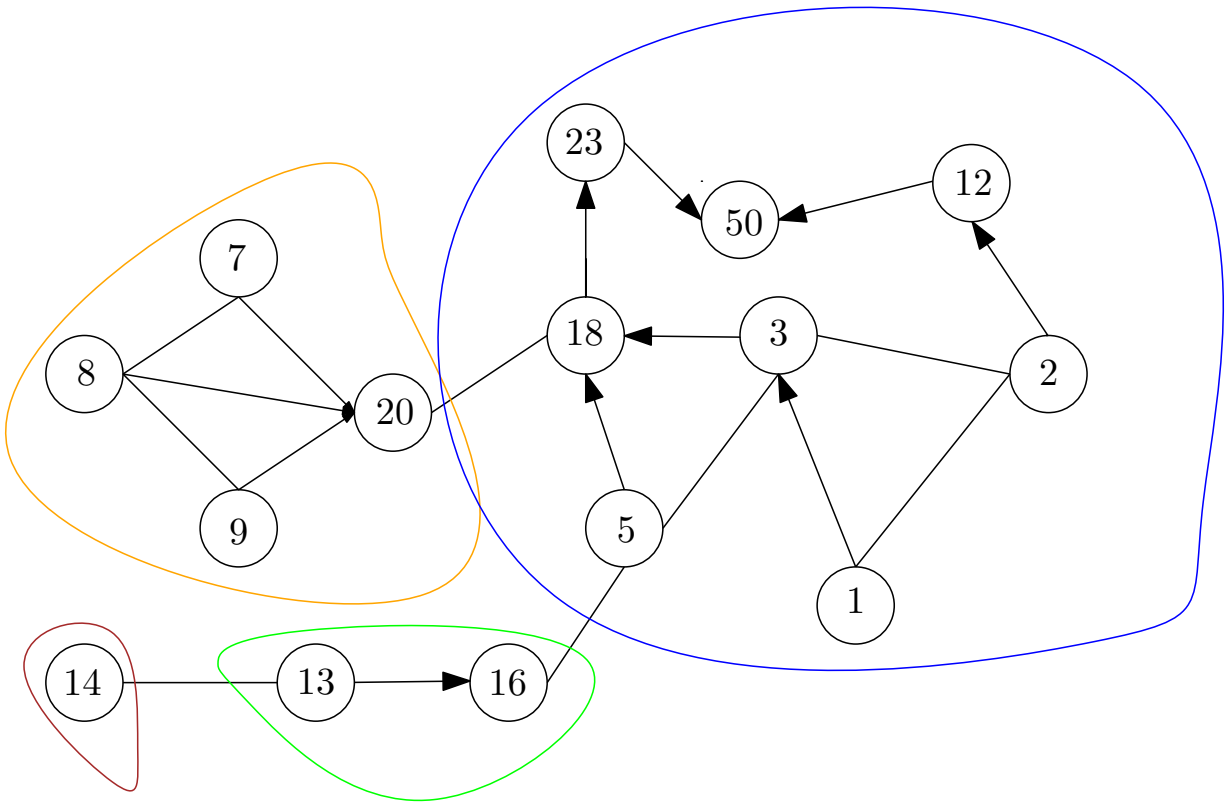

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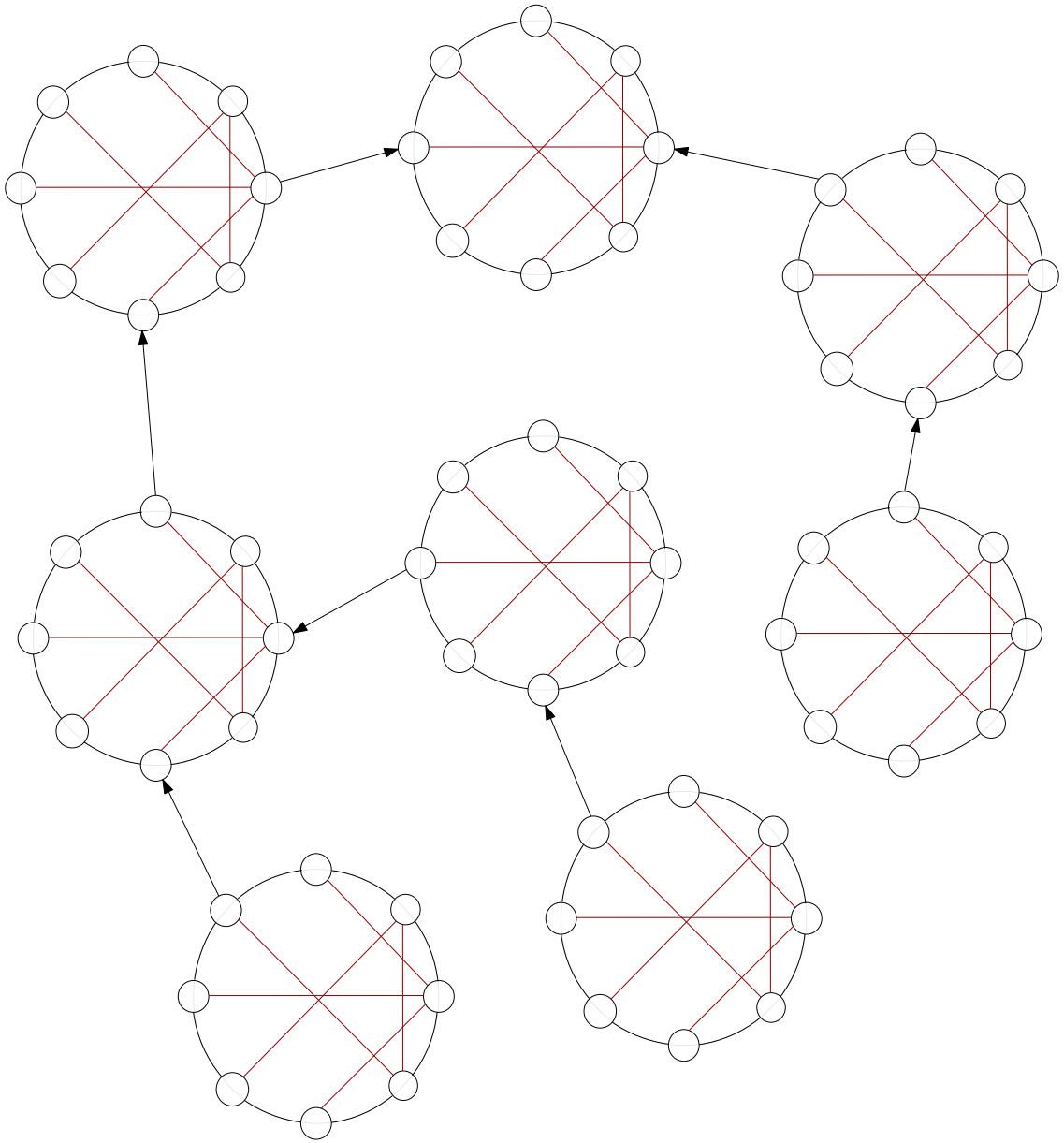

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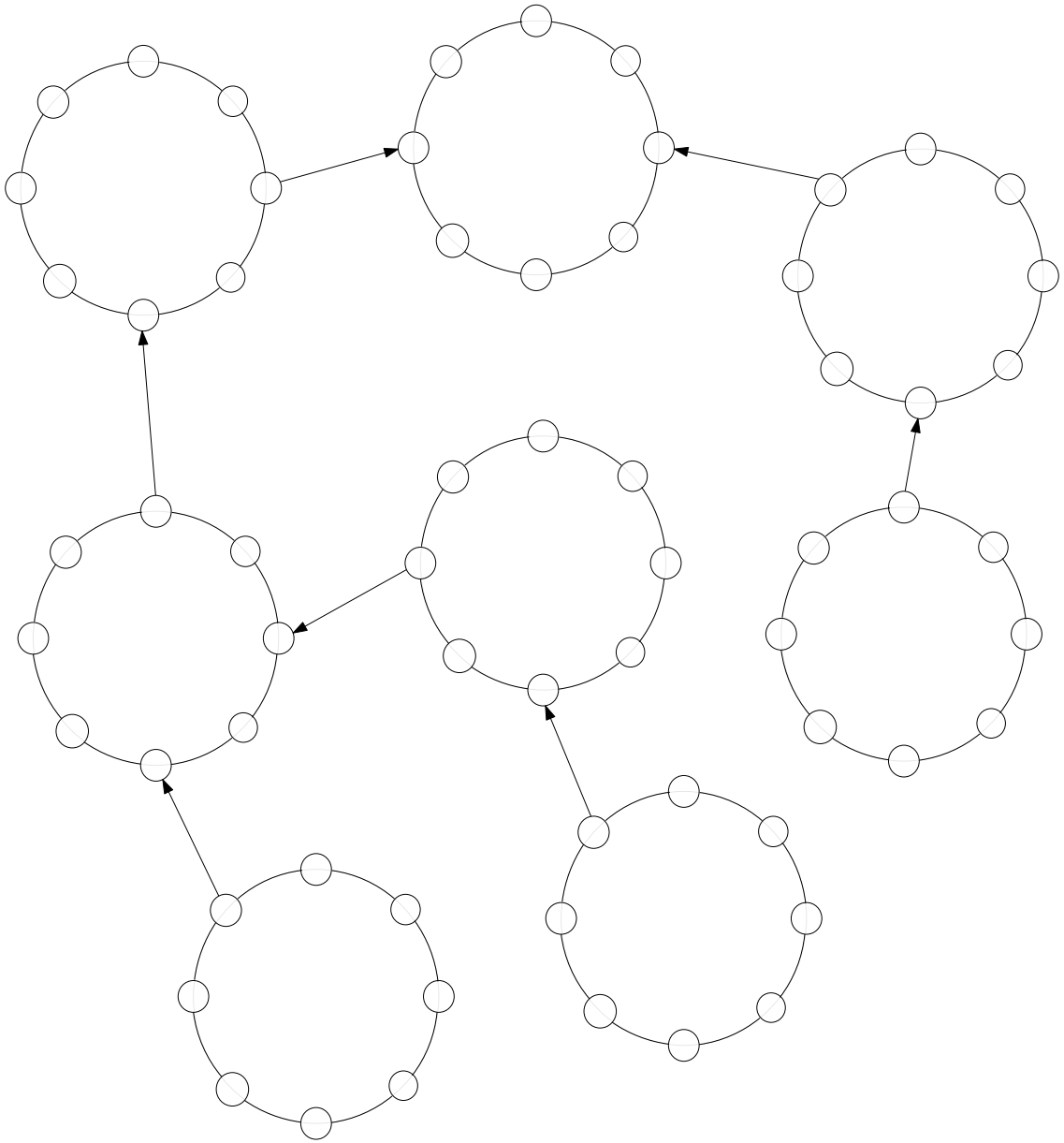

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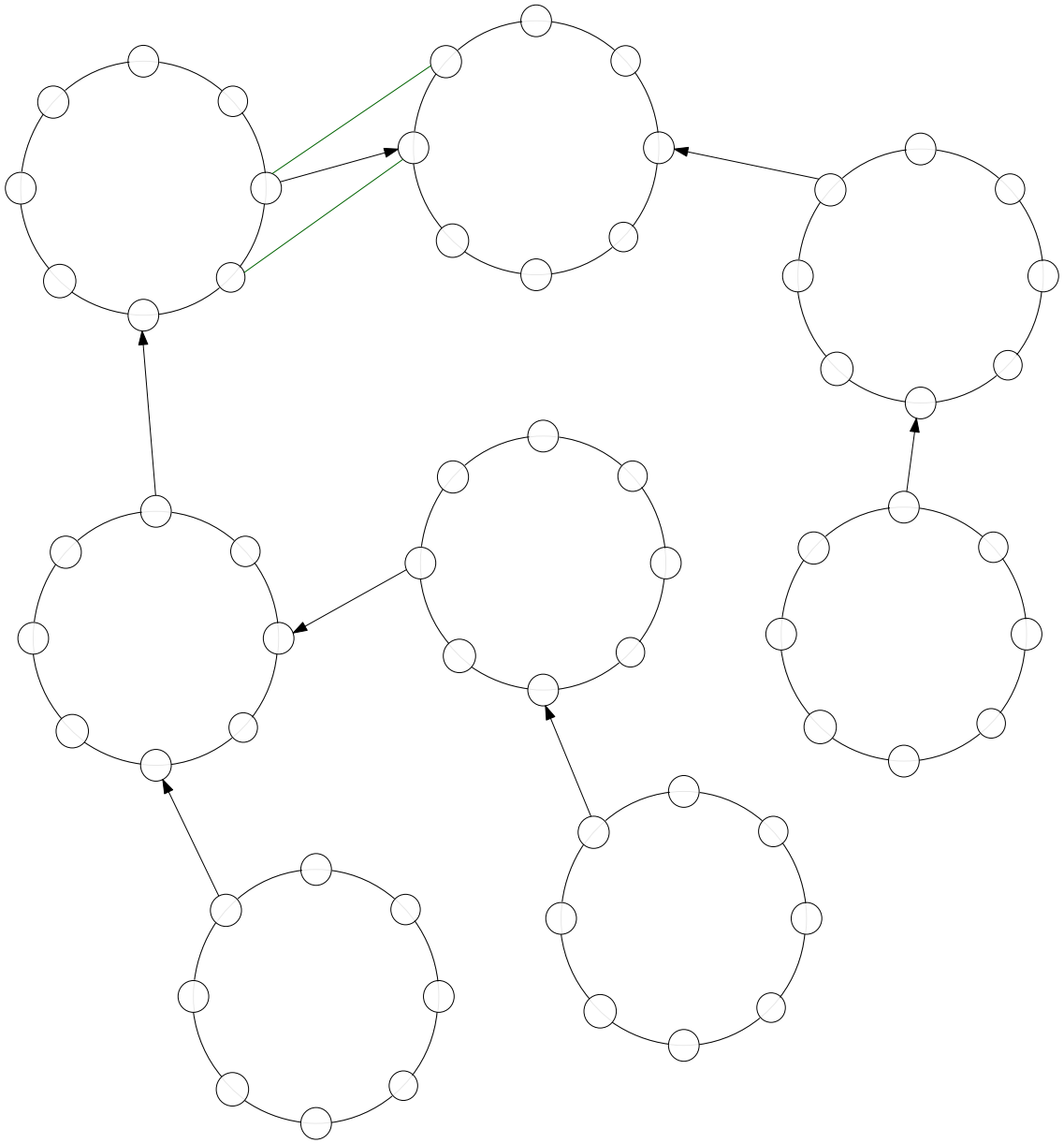

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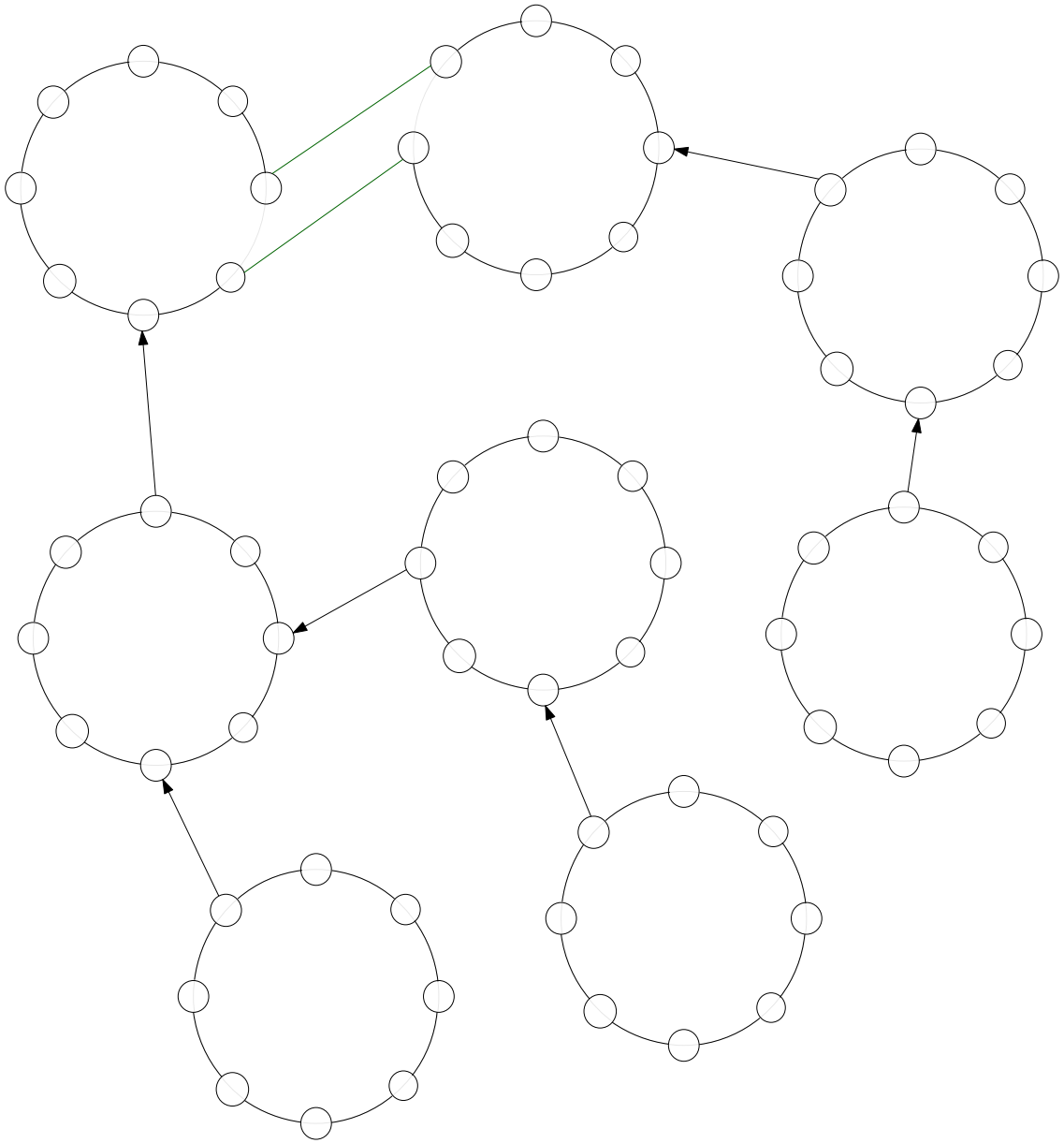

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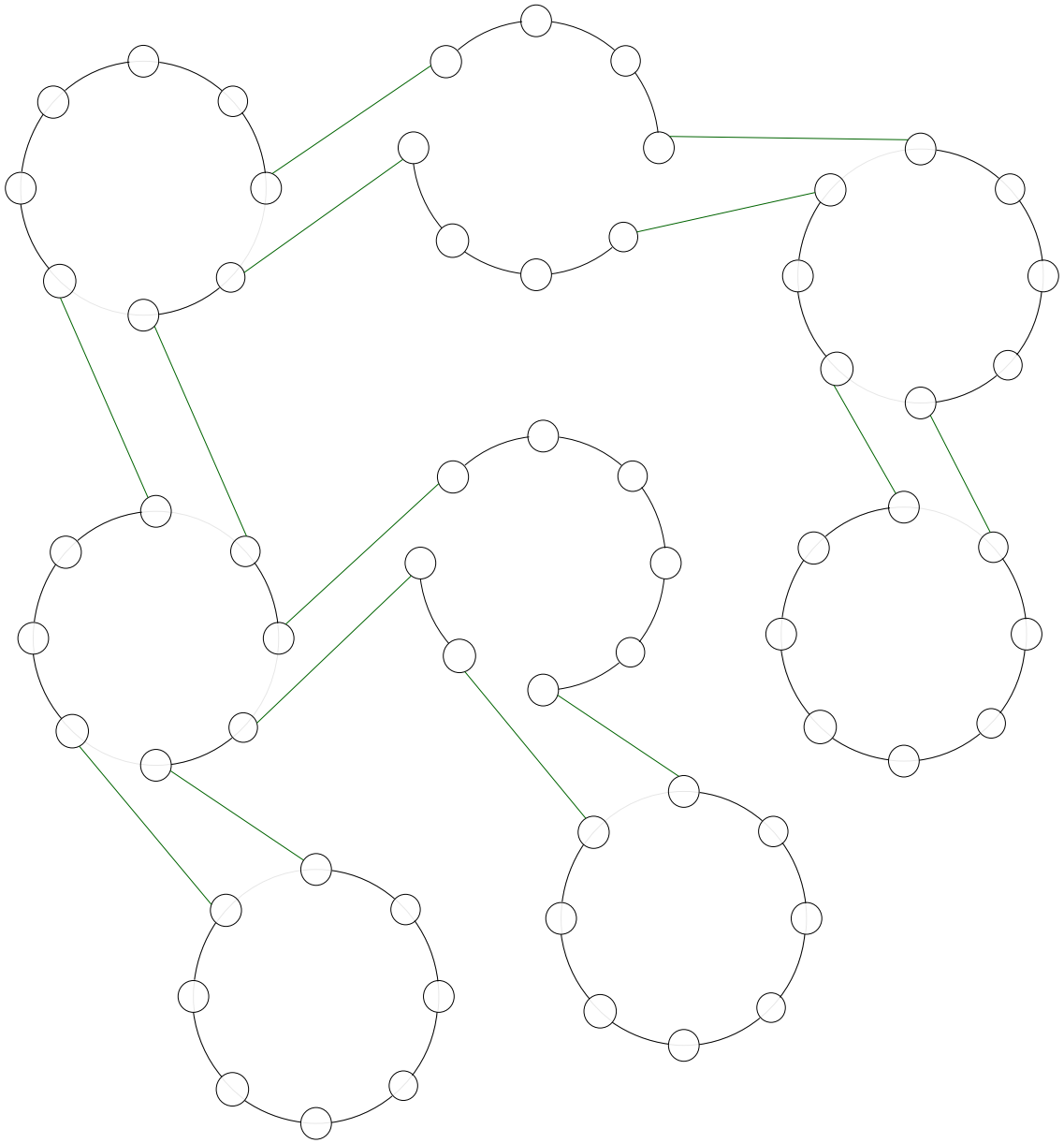

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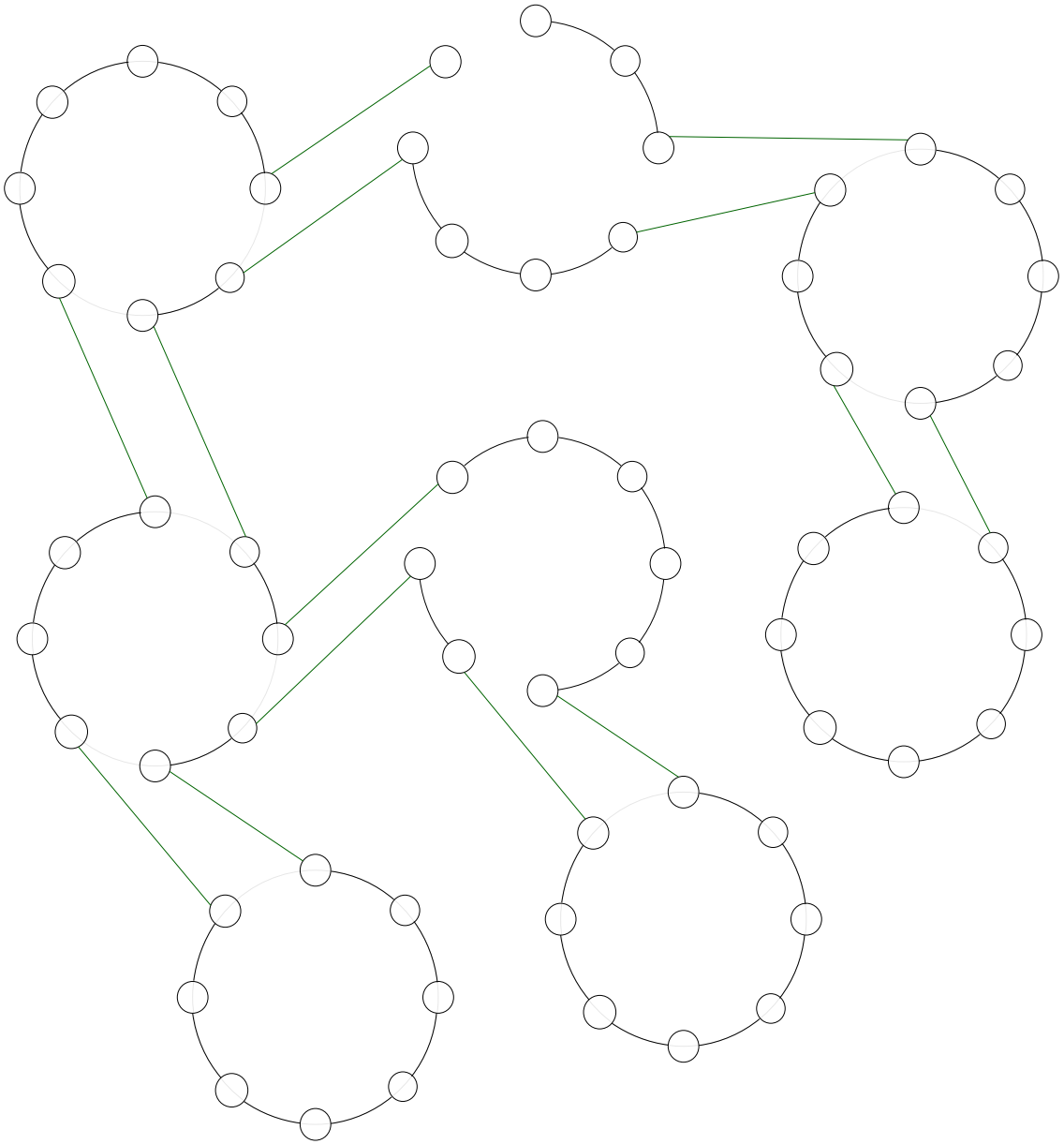

GraphToThinWreath

Theorem: For any initial connected graph with polylogarithmic degree, the GraphToThinWreath algorithm solves Depth-log n Tree in $O(\frac{\log^2 n}{\log \log n})$ time with $O(n \log^2 n)$ total edge activations, $O(n)$ active edges per round and $O(1)$ maximum activated degree

Few Words:

- Gadget network: ThinWreath
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- Requires knowledge of the size of the network
- Changes in low and high level strategy

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GraphToThinWreath

Theorem: For any initial connected graph with polylogarithmic degree, the GraphToThinWreath algorithm solves Depth-log n Tree in $O(\frac{\log^2 n}{\log \log n})$ time with $O(n \log^2 n)$ total edge activations, $O(n)$ active edges per round and $O(1)$ maximum activated degree

Few Words:

- Gadget network: ThinWreath
- LineToCompletePolylogarithmicTree subroutine
- Requires knowledge of the size of the network
- Changes in low and high level strategy

Conclusion

Summary:

- Three Algorithms
- Trade off between time and edge complexity

Open Problems:

- Improve time and degree together
- Change the property of the target network
- Lower Bounds

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Our Results

Algorithm	Time	Total Edge Activations	Maximum Activated Edges	Maximum Activated Degree
GraphToStar	$O(\log n)$	$O(n \log n)$	$O(n)$	$O(n)$
GraphToWreath	$O(\log^2 n)$	$O(n \log^2 n)$	$O(n)$	$O(1)$
GraphToThinWreath	$O(\log^2 n / \log \log n)$	$O(n \log^2 n)$	$O(n)$	$O(\log^2 n)$

Table: Algorithms for Depth-d Tree

Bounds	Time	Total Edge Activations	Maximum Activated Edges	Maximum Activated Degree
Centralized Lower	$\Omega(\log n)$	$\Omega(n)$	$\Omega(n / \log n)$	
Centralized Upper	$O(\log n)$	$\Theta(n)$		
Distributed Lower	$O(\log n)$	$\Omega(n \log n)$		

Table: Bounds for Depth-d Tree

Thank you!!!