Knights, Knaves, and Logical Reasoning

Mechanising the Laws of Thought

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The University of Manchester

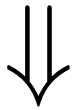
15 July 2015

[1] Special thanks to Francis Southern

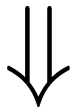
Introduction

Thinking

Formalising



Modelling



Computing

Thinking

A Puzzle

You are on a strange island where people are divided into

- Knights – always saying the truth
- Knaves – always saying lies

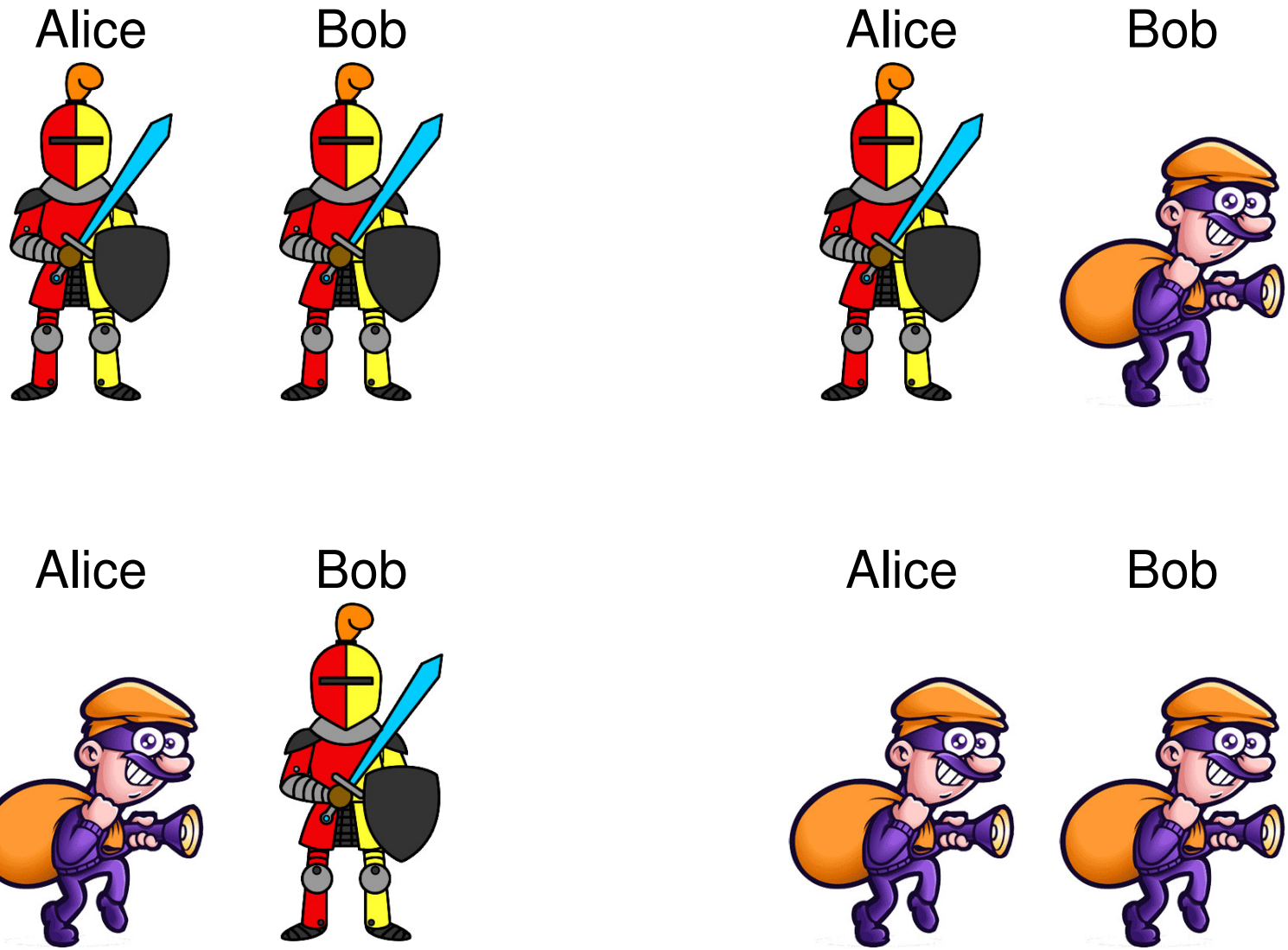
You meet two natives of the island Alice and Bob, and ask them

"Are you knights or knaves?"

Alice answers *"At least one of us is a knave"*

What are Alice and Bob?

Alice: *"At least one of us is a knave"*



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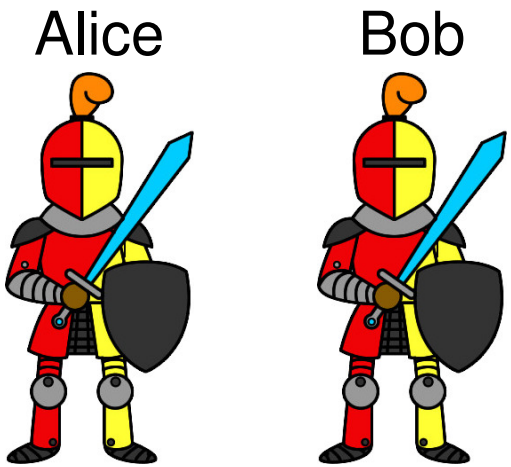

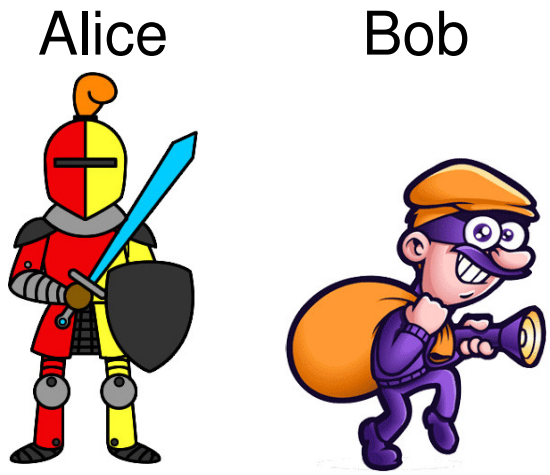

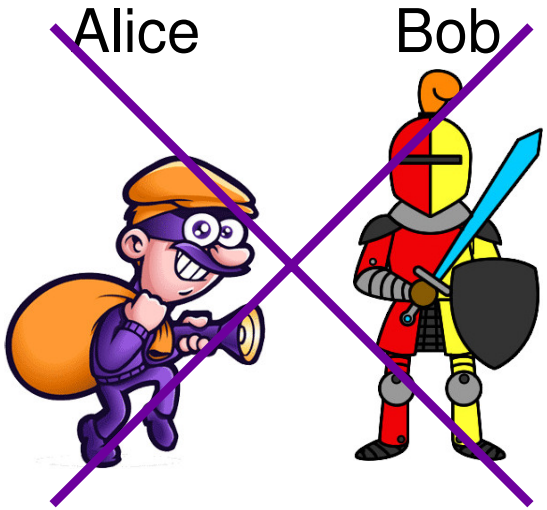

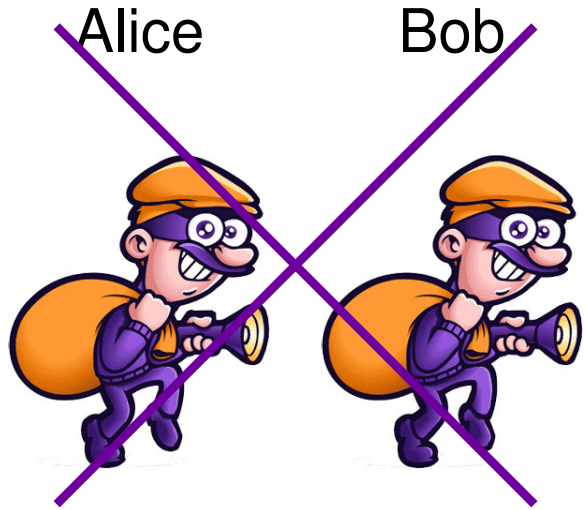

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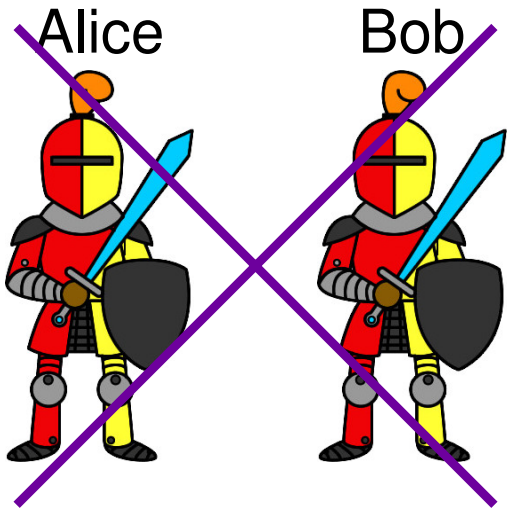

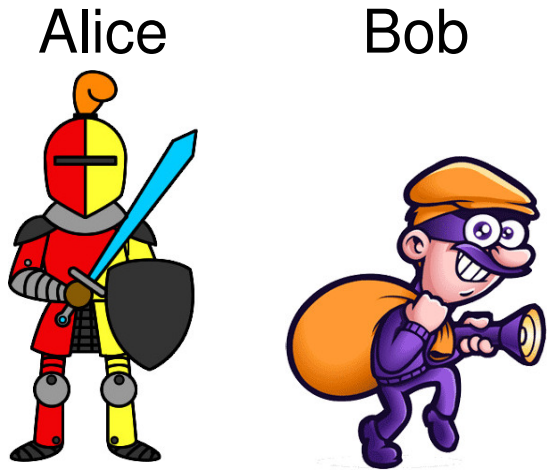

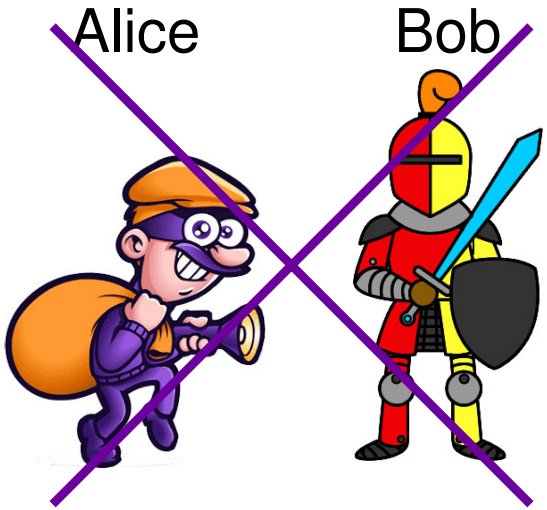

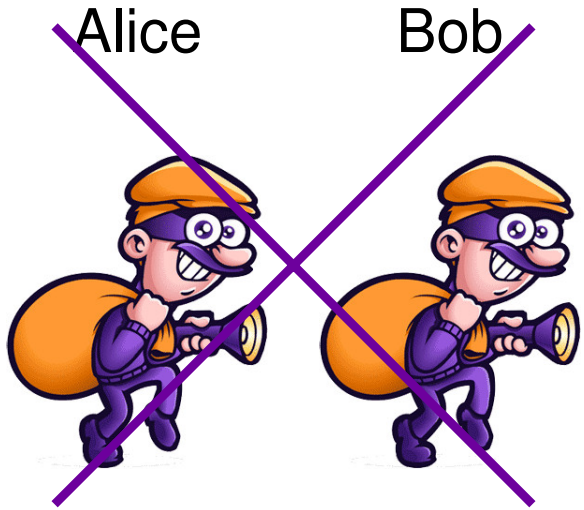

Formalising

Formalising Correct Reasoning

A: Socrates is a man

B: All men are mortal

C: All men are Socrates

C: Socrates is mortal

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Woody Allen - Love and Death

C: Socrates is mortal

Aristotle

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Aristotle

Linguistic, philosophical, or mathematical approaches to formalisation

Today: Propositional Logic

Propositions

An expression which is either true or false.

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An expression which is either true or false.

Proposition test: Is it true that. . . ?

- $2 + 2 = 5$
- Manchester
- Grass is green
- We're in Manchester
- What's your name?
- It's raining

Not – $\neg$, And – $\&$

Not

p	$\neg p$
F	T
T	F

It's *not* raining

Grass is *not* green.

Not – $\neg$, And – $\&$

Not

p	$\neg p$
F	T
T	F

It's *not* raining

Grass is *not* green.

And

p	q	$p \,\&\, q$
F	F	F
F	T	F
T	F	F
T	T	T

Grass is green *and* it's raining.

We're in Manchester *and* we're in France.

Or – $|$, Implication (If, then) – $\rightarrow$

Or

p	q	$p \mid q$
F	F	F
F	T	T
T	F	T
T	T	T

Take an aspirin *or* lie down.

You can have milk *or* sugar in your tea.

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Implication

p	q	$p \rightarrow q$
F	F	T
F	T	T
T	F	F
T	T	T

If you get 90% on this assignment, *then* you'll pass the course.

If you're late, *then* you'll give me a fiver.

Biimplication (If and only if) – $\leftrightarrow$

Biimplication

p	q	$p \leftrightarrow q$
F	F	T
F	T	F
T	F	F
T	T	T

I'll buy you a new wallet *if (and only if)* you need one.

He studies *if (and only if)* he can.

An Example: $(p \,\&\, q) \to r$

p	q	r	$(p \,\&\, q)$	$(p \,\&\, q) \to r$
F	F	F		
F	F	T		
F	T	F		
F	T	T		
T	F	F		
T	F	T		
T	T	F		
T	T	T		

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F	F	F	F	
F	F	T	F	
F	T	F	F	
F	T	T	F	
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F	F	T	F	T
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Modelling

The Trick

k_A = Alice is a knight $\qquad$ $\neg k_A$ = Alice is a knave

"Alice says X" is the same as $k_A \leftrightarrow X$

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Alice says

- "at least one of us is a knave"
- "I'm a knave or Bob is a knave"
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$$\Rightarrow k_A \leftrightarrow (\neg k_A \mid \neg k_B)$$

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F	F	T	T	T	
F	T	T	F	T	
T	F	F	T	T	
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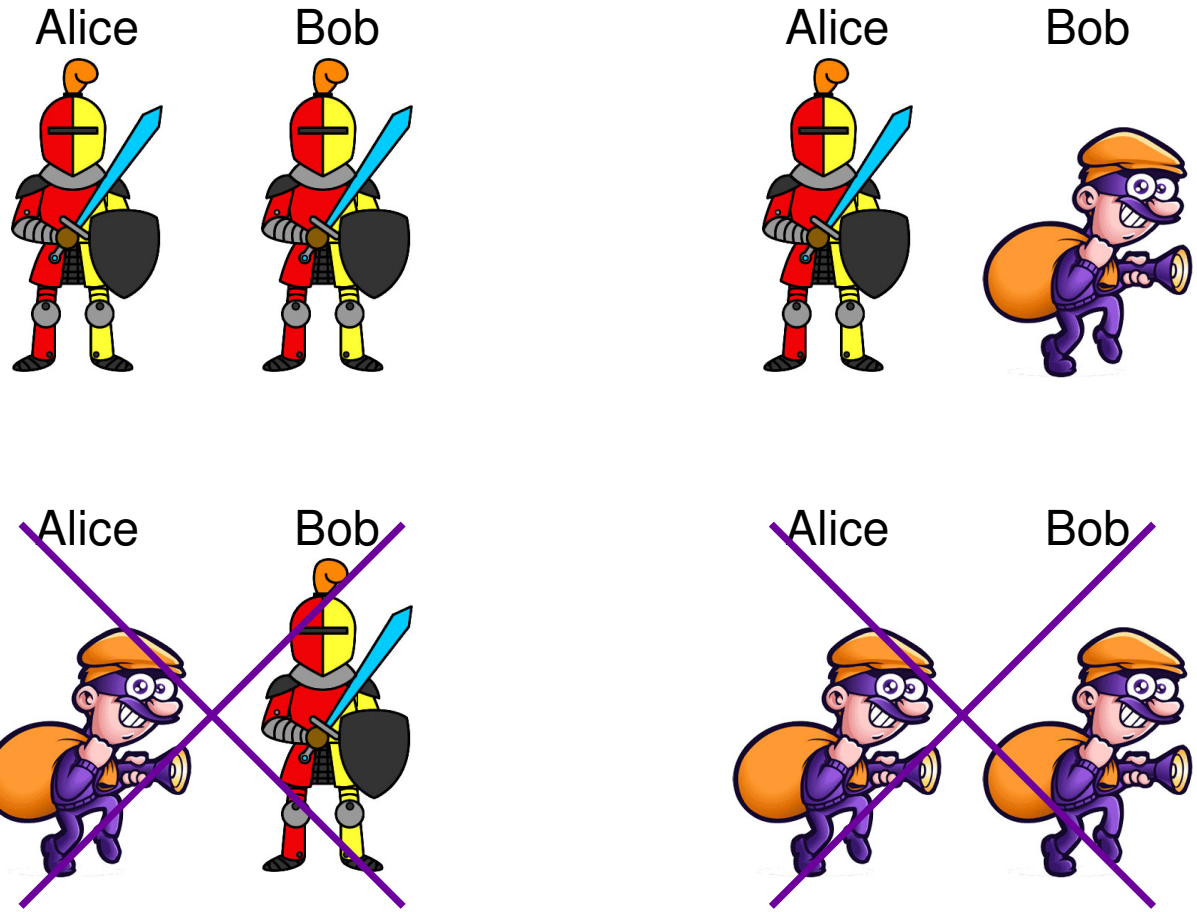
From Solving to Modelling

Alice: *"At least one of us is a knave"*



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It can be (really) hard, but you only have to do it once!

Modelling a Sudoku

			7			4	1	
		3		2				6
1		7	4			5	2	3
4		1	6				8	
	2	9		7		6	3	
	7				4	2		1
7	5	2			6	3		9
3				4		1		
	1	4			3			

What propositions do we need?

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Number n is in row i and column j

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$$p_{1,1,1},\ p_{1,1,2},\ \ldots,\ p_{9,9,8},\ p_{9,9,9}$$

729 propositions!

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What do we have to model?

- at least one number per cell $(p_{1,1,4} \mid \ldots \mid p_{9,1,4})$
- at most one number per cell $(p_{7,1,4} \rightarrow \neg p_{1,1,4},\ p_{7,1,4} \rightarrow \neg p_{2,1,4})$

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- no number can be repeated in a row

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- no number can be repeated in a row/column

Modelling a Sudoku

			7			4	1	
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- no number can be repeated in a row/column/region

Computing

Automating the Process

Truth table

- mechanical
- time consuming (2^n rows!)
- tedious

Automating the Process

Truth table

- mechanical
- time consuming (2^n rows!)
- tedious

Let a computer do it for you!

- ideal for mechanical tasks
- only needs an input formula
- more reliable than us
- much faster than us
- the output is easily customisable

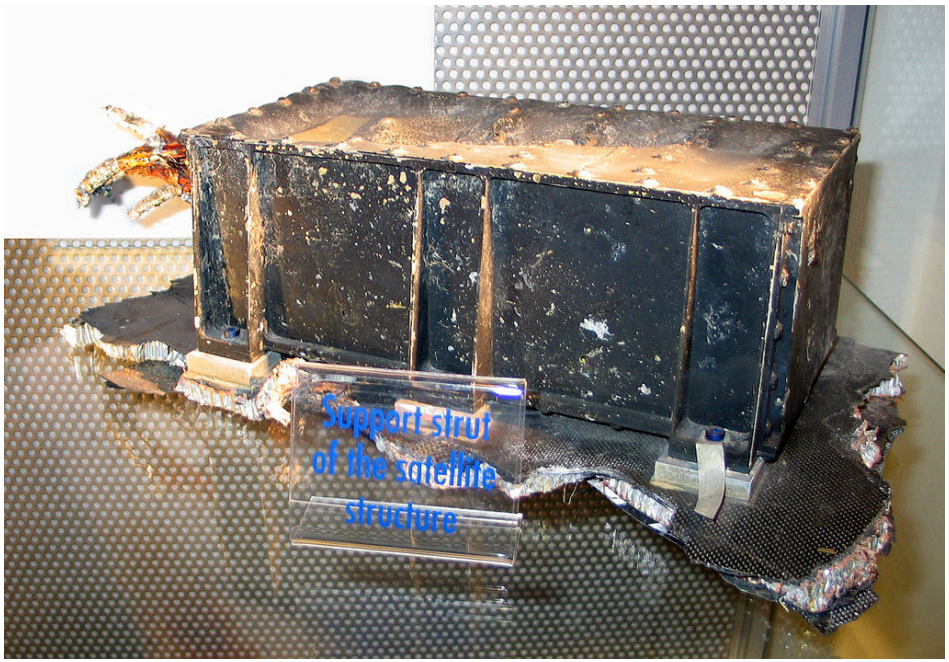
Automated Reasoning

Much more than solving puzzles!

- software and hardware verification
  Intel and Microsoft
- information management
  biomedical ontologies, Semantic Web, databases
- combinatorial reasoning
  constraint satisfaction, planning, scheduling
- Internet security
- theorem proving in mathematics

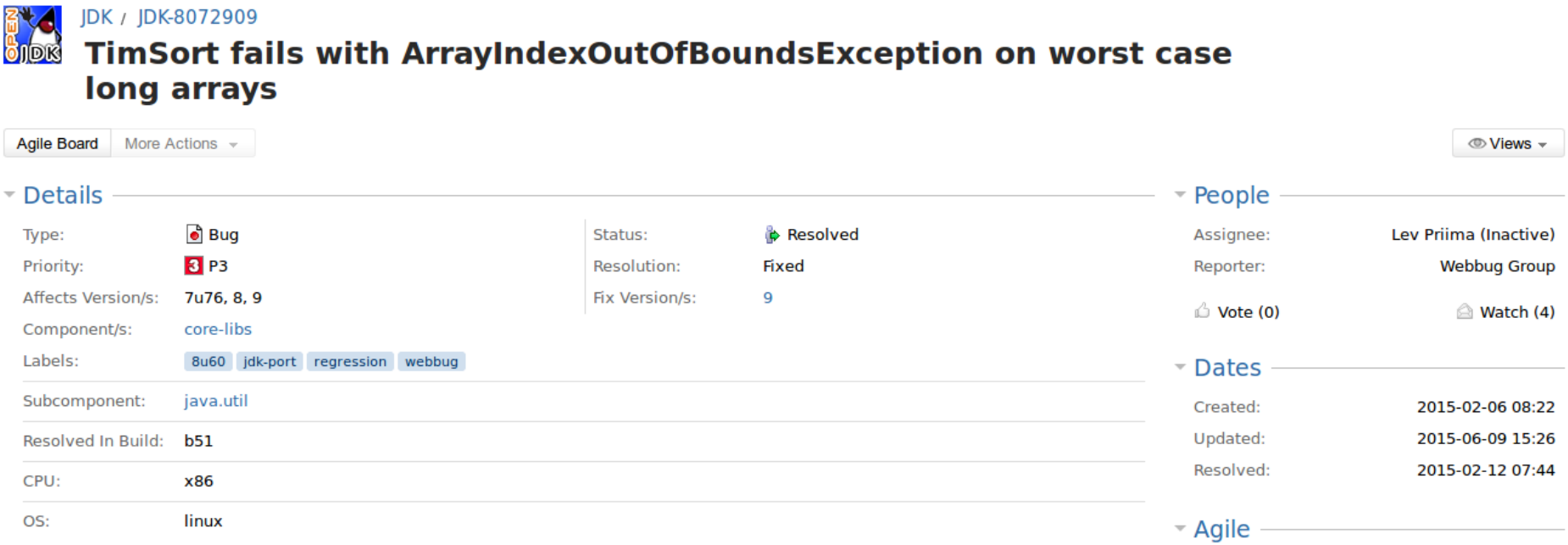
Where Could Have Been Used

Ariane 5 rocket failure due to a software bug, cost $370 million.



Where Has Been Used

To find and fix a bug in a widely used sorting algorithm!



JDK / JDK-8072909

TimSort fails with ArrayIndexOutOfBoundsException on worst case long arrays

Agile Board | More Actions | Views

Details

Type:	Bug	Status:	Resolved
Priority:	P3	Resolution:	Fixed
Affects Version/s:	7u76, 8, 9	Fix Version/s:	9
Component/s:	core-libs		
Labels:	8u60 jdk-port regression webbug		
Subcomponent:	java.util		
Resolved In Build:	b51		
CPU:	x86		
OS:	linux		

People

Assignee:	Lev Priima (Inactive)
Reporter:	Webbug Group
Vote (0)	Watch (4)

Dates

Created:	2015-02-06 08:22
Updated:	2015-06-09 15:26
Resolved:	2015-02-12 07:44

Agile

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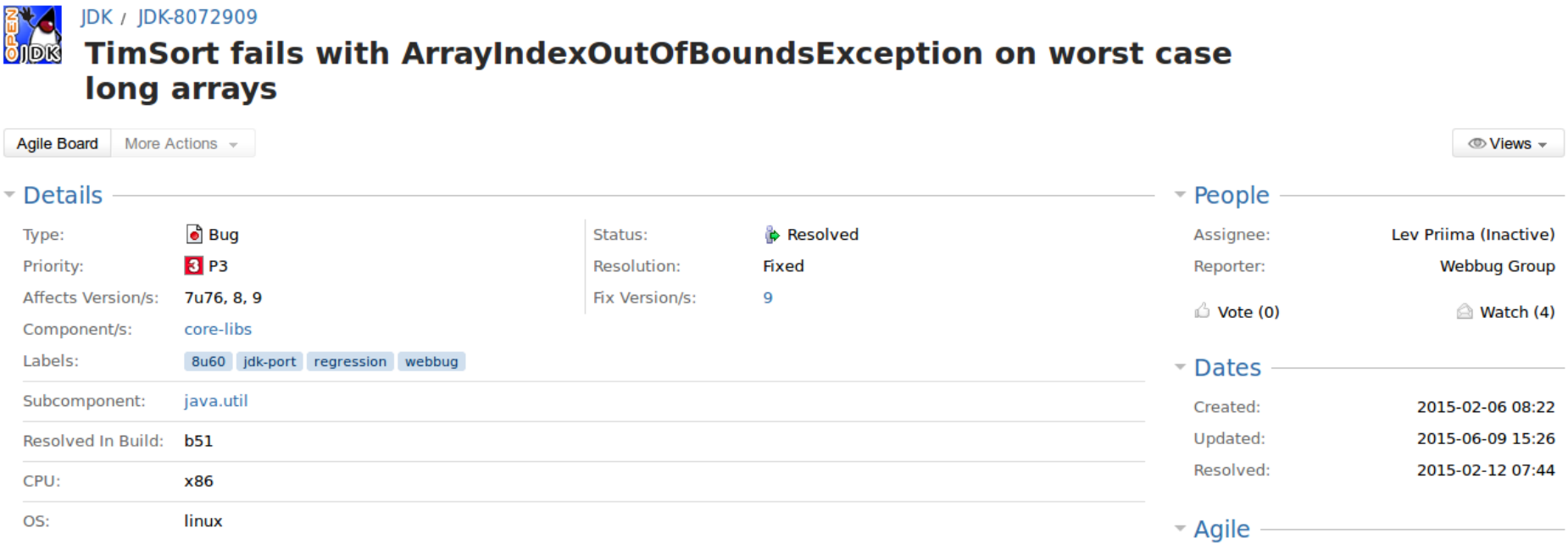

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Agile

Even Amazon and Facebook use automated reasoning techniques!

Automated Reasoning Competitions

- The CADE ATP System Competition (CASC)
- OWL Reasoning Competition (ORE)
- SAT-Race

Automated Reasoning Competitions

- The CADE ATP System Competition (CASC)
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- SAT-Race

You can bet on the winner!

Do You Want to Know More?

Look at the references on the handout!