Knights, Knaves, and Logical Reasoning

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1 Puzzles

In these exercises, you have met two natives (called, imaginatively, A and B) and you wish to establish as much information as you can about their status as knights or knaves. Remember, everything a knight says is true and everything a knave says is false.

1. A says "We are both knaves".
    - A is a □ Knight - □ Knave
    - B is a □ Knight - □ Knave
2. A says "If I am a knight, then so is B".
    - A is a □ Knight - □ Knave
    - B is a □ Knight - □ Knave

2 Use the Trick!

Fill the truth table and solve the puzzles!

1. A says "We are both knaves"

   Using the trick it becomes $k_A \leftrightarrow$__________

k_A	k_B	____	____	___________	$k_A \leftrightarrow$ ___________
F	F				
F	T				
T	F				
T	T				

[*] Special thanks to Francis Southern, who contributed significantly to this material!

2. A says “If I am a knight, then so is B”

   Using the trick it becomes $k_A \leftrightarrow$ ________

k_A	k_B	________	$k_A \leftrightarrow$ ________
F	F		
F	T		
T	F		
T	T		

3 Sudoku

			7			4	1	
		3		2				6
1		7	4			5	2	3
4		1	6				8	
	2	9		7		6	3	
	7				4	2		1
7	5	2			6	3		9
3				4		1		
	1	4			3			

Fabio’s time: 6:54

Computer time: 0.006998 seconds

Your time: _____

4 References

- I recommend heavily everything by Raymond Smullyan. In preparing this lesson we referred to his puzzle books *What Is the Name of This Book? The Riddle of Dracula and Other Logical Puzzles*, *Forever Undecided* and *The Lady or the Tiger?*.
- `www.cut-the-knot.org` and `www.khanacademy.org/` have sections on knight and knave puzzles, as well as a huge number of other mathematical topics.
- Or you can simply look up *propositional logic* on Google or Wikipedia.
- For more advanced reading, Smullyan has a more academic book called *First-order Logic*, which is quite a step up in difficulty from what we’ve done today, but excellent nevertheless. A slightly more gentle, but still very rigorous book, is *A Mathematical Introduction to Logic* by Herbert Enderton. Of course, there are many other possibilities out there.

- Another more advanced source is the Stanford Encyclopaedia of Philosophy (not just of interest to philosophers!). An interesting starting place might be `http://plato.stanford.edu/entries/reasoning-automated/`.

- A classic informal introduction to formal systems *Gödel, Escher, Bach: An Eternal Golden Braid* by Douglas Hofstadter.

- Competitions

  CASC `http://www.cs.miami.edu/~tptp/CASC/`

  ORE (this year web-page)

  `http://www.w3.org/community/owled/ore-2015-workshop/competition/`

  SAT-race `http://www.satcompetition.org/`

- MiniSat `http://minisat.se/MiniSat.html`

- Truth Table Generator

  `http://programming.dojo.net.nz/study/truth-table-generator/index`

- Well-known software bugs that could have been avoided using automated reasoning techniques `http://en.wikipedia.org/wiki/List_of_software_bugs`.

  (Ariane 5 `http://en.wikipedia.org/wiki/Cluster_(spacecraft)`)

- Proving that Androids, Javas and Pythons sorting algorithm is broken (and showing how to fix it)

  `http://envisage-project.eu/proving-android-java-and-python-sorting-algorithm-is-broken-and-how-to-fix-it/`

- Amazon uses formal methods! `http://research.microsoft.com/en-us/um/people/lamport/tla/formal-methods-amazon.pdf`

- Facebook uses formal methods! `https://research.facebook.com/publications/422671501231772/moving-fast-with-software-verification/`