A Modal Tableau Approach for Minimal Model Generation

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Aim

To create a tableau calculus for modal formulae which:

- works on a set of modal clauses
- generates only minimal modal Herbrand models
- generates all minimal modal Herbrand models
- generates each minimal modal Herbrand model only once
- terminates

(Minimal) Modal Herbrand Model

Modal Herbrand universe ($W_{\mathcal{U}}$): The set of all ground terms built from a fixed constant w and a supply of unary function symbols $f_{\Diamond\phi}$ uniquely associated with subformulae $\Diamond\phi$ of a modal formula φ.

Modal Herbrand interpretation: A set composed of labelled formulae of the form $u : p$ and labelled relations of the form $(u, v) : R$.

Modal Herbrand model: If a modal Herbrand interpretation I is such that $I \models w : \varphi$, then I is a *modal Herbrand model* of φ.

Example: a modal Herbrand model for $p_1 \wedge (\Diamond p_2 \vee p_3)$ is

$$\{w : p_1, w : p_3, f_{\Diamond p_2}(w) : p_2, (w, f_{\Diamond p_2}(w)) : R\}$$

Minimal modal Herbrand model: A modal Herbrand model I of a modal formula φ is a *minimal modal Herbrand model* iff every other modal Herbrand model I' of φ, if $I' \subseteq I$ then $I = I'$.

Example: the minimal modal Herbrand models for $p_1 \wedge (\Diamond p_2 \vee p_3)$ are:

$I_1 = \{w : p_1, w : p_3\}$ and $I_2 = \{w : p_1, f_{\Diamond p_2}(w) : p_2, (w, f_{\Diamond p_2}(w)) : R\}$

Minimal Modal Model Generation Calculus

Input: a set of modal clauses $\xrightarrow[\text{during the CNF transformation}]{\textbf{Box miniscoping}}$ **Box miniscoping:** $\Box(\phi_1 \wedge \phi_2) \Rightarrow \Box\phi_1 \wedge \Box\phi_2$
a conjunction appears only in the scope of a diamond operator

Expansion strategy: depth-first left-to-right strategy. Without this strategy the calculus is no longer minimal model sound and complete.

Two possible **outputs:**

- the input is unsatisfiable (closed tableau)
- all and only minimal modal Herbrand models, each model exactly once (fully expanded open tableau: open branch $\rightsquigarrow$ a minimal model)

Expansion rules

$$(\Diamond)\ \frac{u : \Diamond(\phi_1 \wedge \ldots \wedge \phi_n)}{\begin{array}{c}(u, f_{\Diamond\phi}(u)) : R\\ f_{\Diamond\phi}(u) : \phi_1\\ \vdots\\ f_{\Diamond\phi}(u) : \phi_n\end{array}}$$

where $\phi = \phi_1 \wedge \ldots \wedge \phi_n$ and $f_{\Diamond\phi}$ is function symbol uniquely associated with $\Diamond\phi$

$$(\vee)_E\ \frac{(u : \phi_1 \vee \ldots \vee \phi_n) \vee \Phi}{(u : \phi_1) \vee \ldots \vee (u : \phi_n) \vee \Phi}$$

$$(CS)\ \frac{\mathcal{P}_1 \vee \ldots \vee \mathcal{P}_n}{\begin{array}{c|c|c|c}\mathcal{P}_1 & \mathcal{P}_2 & & \mathcal{P}_n\\ neg(\mathcal{P}_2) & \vdots & \iddots & \\ \vdots & neg(\mathcal{P}_n) & & \\ neg(\mathcal{P}_n) & & & \end{array}}$$

$$\begin{array}{ccc}u_1 : p_1 & \ldots & u_n : p_n\\ (v_1, w_1) : R & \ldots & (v_m, w_m) : R\\ (s_1, t_1) : R & \ldots & (s_j, t_j) : R\end{array}$$

$$(PUHR)\ \frac{\begin{array}{c}u_1 : \neg p_1 \vee \ldots \vee u_n : \neg p_n \vee v_1 : \Box\phi_1 \vee \ldots \vee v_m : \Box\phi_m\\ \vee (s_1, t_1) : \neg R \vee \ldots \vee (s_j, t_j) : \neg R \vee \Psi\end{array}}{(w_1 : \phi_1) \vee \ldots \vee (w_m : \phi_m) \vee \Psi}$$

Model constraint propagation rule

If $H = \{u_1 : p_1, \ldots, u_n : p_n, (v_1, w_1) : R, \ldots, (v_m, w_m) : R\}$ is a model extracted from an open and fully expanded branch $\mathcal{B}$, then the following *model constraint clause*

$$u_1 : \neg p_1 \vee \ldots \vee u_n : \neg p_n \vee (v_1, w_1) : \neg R \vee \ldots \vee (v_m, w_m) : \neg R$$

is added to all the branches to the right of $\mathcal{B}$

$(\Diamond)$ **rule:**

- the union of the standard α rule and diamond rule
- $f_{\Diamond\phi}(u)$ is a Skolem term uniquely associated with the premise

$(\vee)_E$ **rule:** switches from labelled disjunction to disjunction of labelled formulae

(CS) **(complement splitting) rule:**

- avoids the creation of a model more than once
- ensures that the first model is minimal

Negation of positive tableau literal is defined by the neg function:

$$neg(\mathcal{P}) = \begin{cases} u : \neg p_i & \text{if } \mathcal{P} = u : p_i\\ (u, v) : \neg R & \text{if } \mathcal{P} = (u, v) : R\\ (u, f_{\Diamond\phi}(u)) : \neg R & \text{if } \mathcal{P} = u : \Diamond\phi. \end{cases}$$

$(PUHR)$ **rule:**

- is the simultaneous application of the closure rules (for labelled formulae and labelled relations) and the box rule
- expands a disjunction of tableau literals where some of the tableau literals are negative iff it is necessary

Model constraint propagation rule: prevents the generation of non-minimal models