Trace Inclusion for One-Counter Nets Revisited

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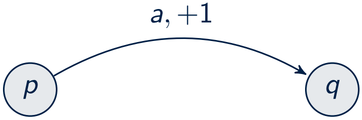
September 23, 2014

One-Counter Automata

$(Q, \mathsf{Act}, \delta)$ $\delta \subseteq (Q \times \mathsf{Act} \times \{-1, 0, +1, =0\} \times Q)$



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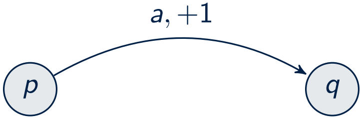
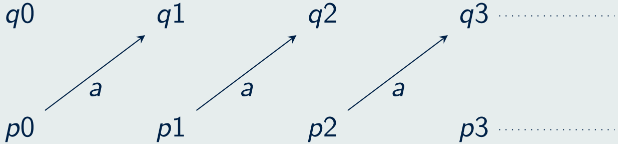


Induced LTS over $Q \times \mathbb{N}$



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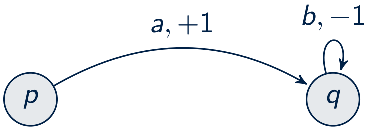

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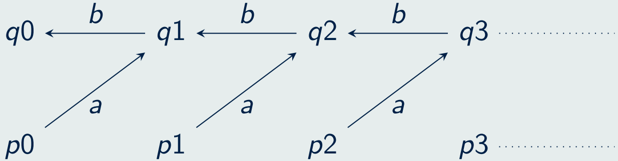

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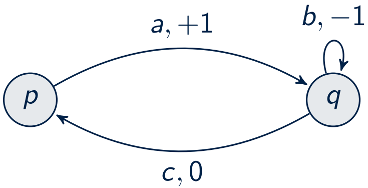

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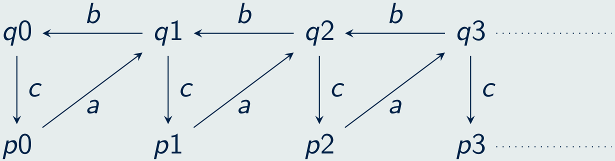

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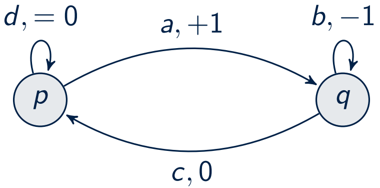

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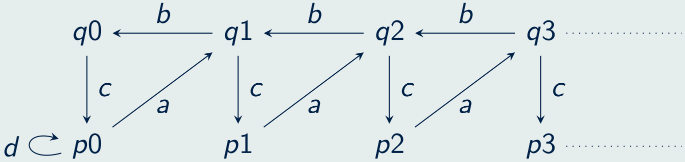

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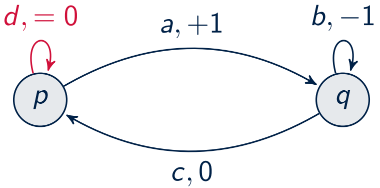

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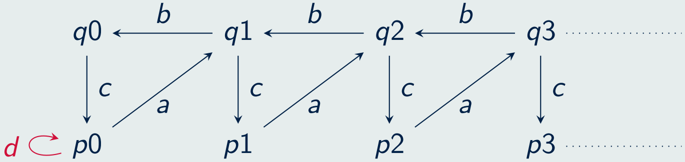

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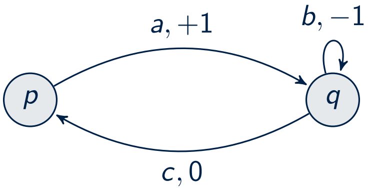

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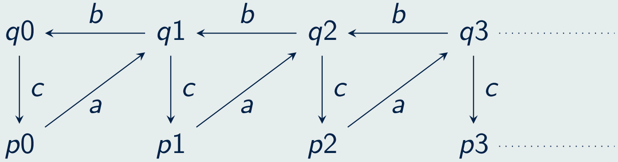
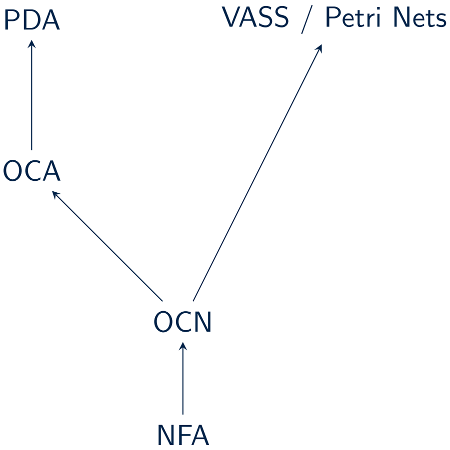


OCN and Related Models



Trace Inclusion for One-Counter Automata

$OCA \subseteq OCA$

INPUT:

- OCA $\mathcal{A}$ and configuration pm
- OCA $\mathcal{A}'$ and configuration $p'm'$

OUTPUT:

yes iff $pm \subseteq p'm'$

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Normal-Form Assumption

Reduction

$$\text{OCA} \subseteq \text{OCA}$$

$$\rightsquigarrow$$

$$\text{non-blocking DOCA} \subseteq \text{complete OCA}$$

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- reduction works in logspace and preserves determinisim

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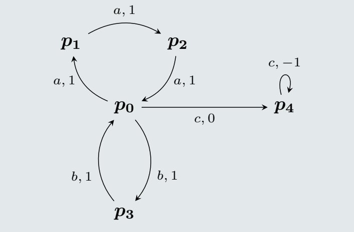
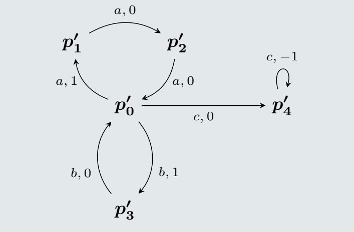
Loops in the synchronous product

$\mathcal{A}$



$\mathcal{A}'$



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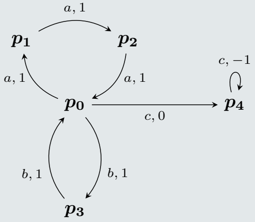

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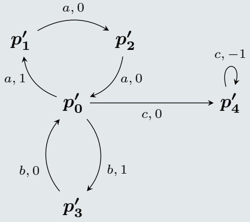
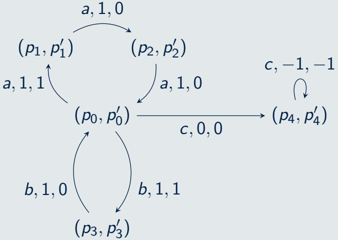


$\mathcal{A} \times \mathcal{A}'$



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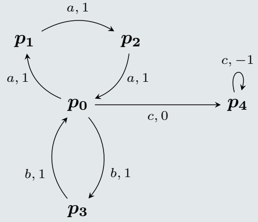

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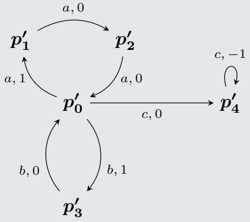

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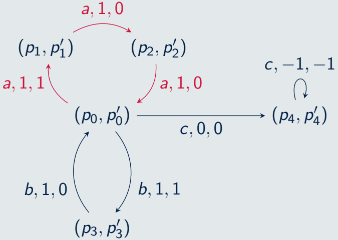

- L_0 is a *loop* with effect $(3, 1)$.

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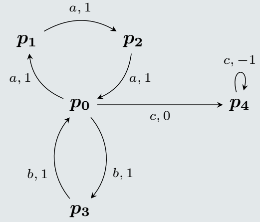

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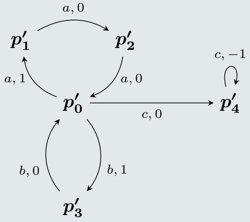

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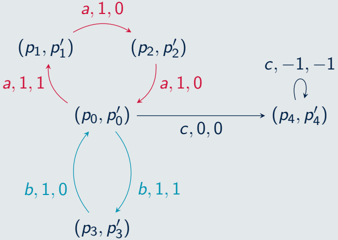

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Loops in the synchronous product

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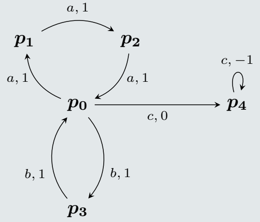

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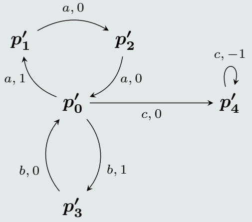

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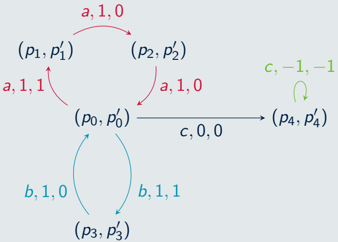
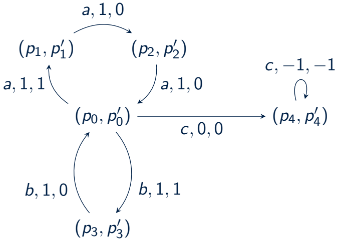


- L_0 is a *loop* with effect $(3, 1)$.
- L_1 is a loop with effect $(2, 1)$.
- L_2 is a loop with effect $(-1, -1)$.

Example: Witnesses for $p_0 0 \not\sqsubseteq p'_0 5$



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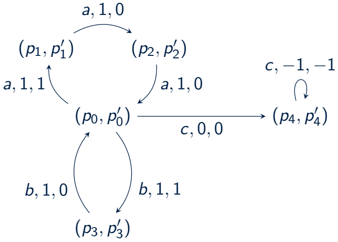

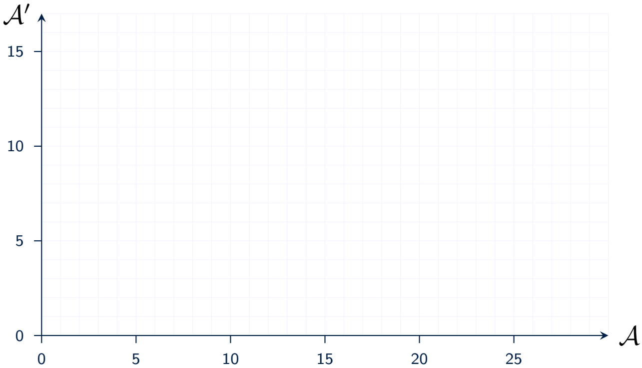

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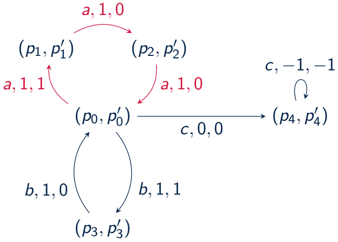
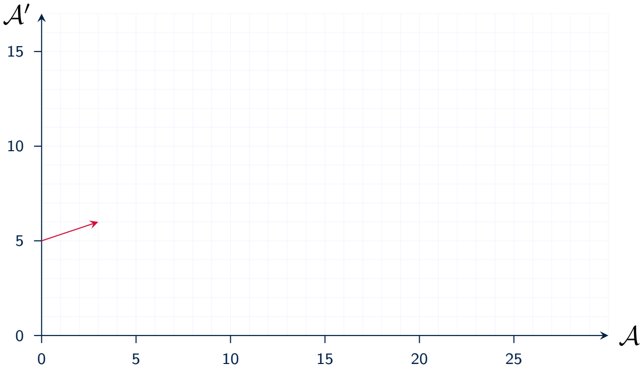


$\pi_0 = (aaa)$



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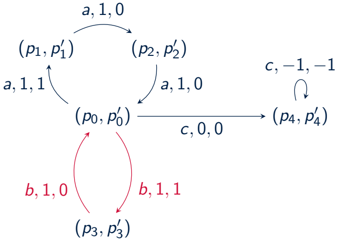
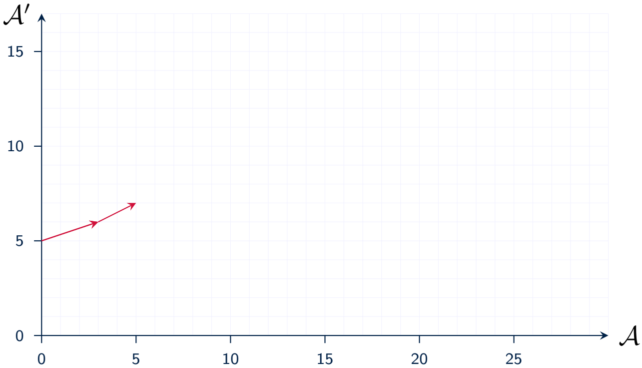


$\pi_0 = (aaa)(bb)$



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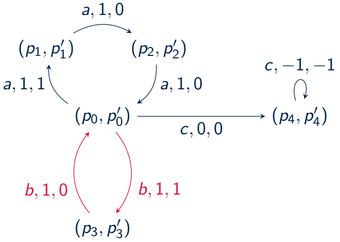
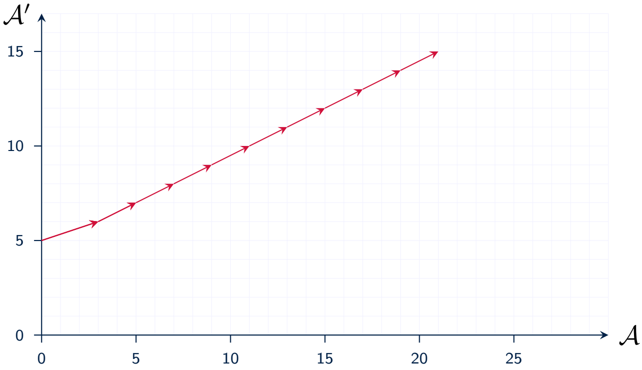


$\pi_0 = (aaa)(bb)^9$



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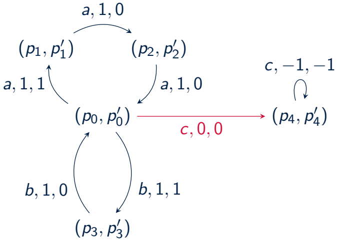
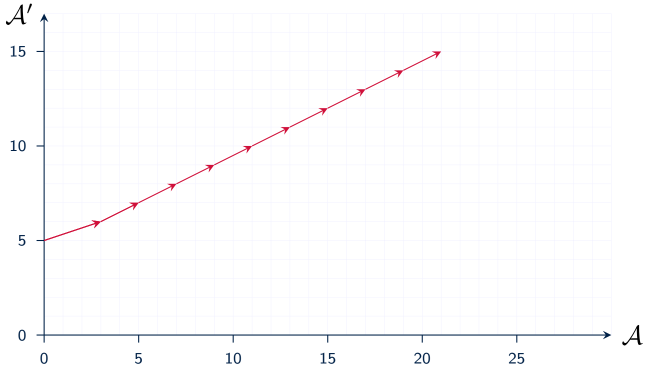


$$\pi_0 = (aaa)(bb)^9 c$$



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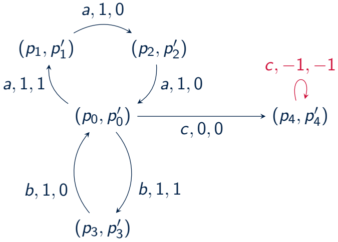
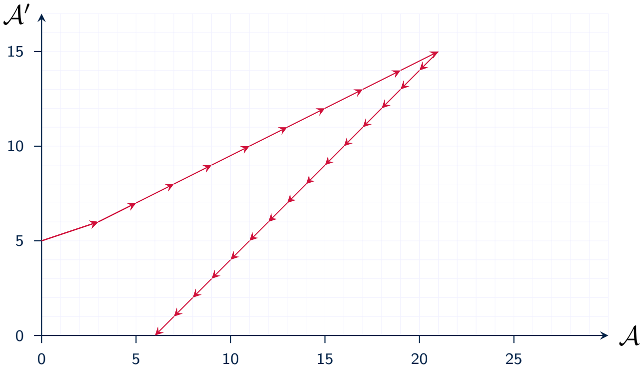


$\pi_0 = (aaa)(bb)^9 cc^{15}$



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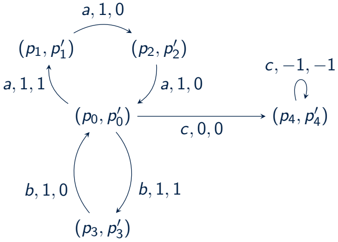

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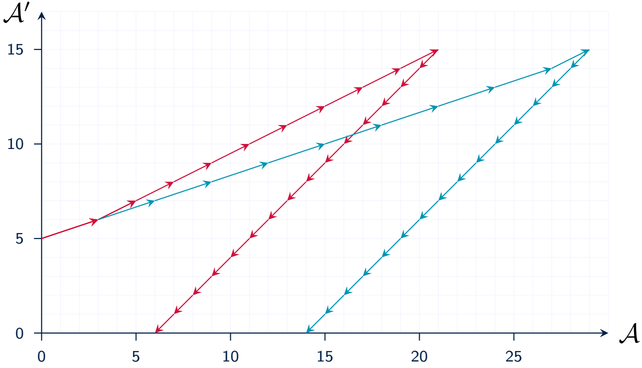

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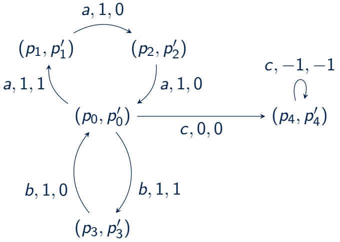

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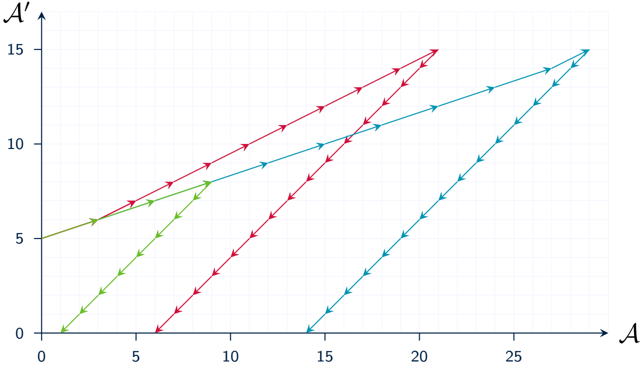

Characterizing Witnesses

Idea

Stepwise rewrite witnesses to “better” ones such that

1. the *loop-structure* is the same.
2. the effect on $\mathcal{A}'$ is the same,
3. the effect on $\mathcal{A}$ does not decrease,
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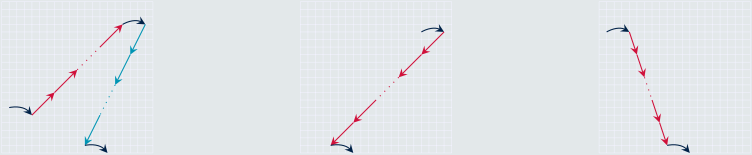
$\rightsquigarrow$ unique normal form for each witness

Characterizing Witnesses

Theorem

If $pm \not\subseteq p'm'$ then there is a short witness, or one of forms



Here, $\rightsquigarrow$ are short paths and $\rightarrow$, $\rightarrow$ are loops that may occur often.

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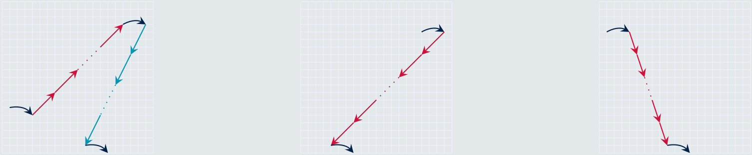

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Solving $DOCN \not\subseteq DOCN$ in NL

- guess short components of a witness $\pi = \pi_0 L_0^{l_0} \pi_1 L_1^{l_1} \pi_2$
- compute and memorize their effects
- check existence of coefficients $l_0, l_1 \in \mathbb{N}$ such that both $m + \Delta(\pi) \geq 0$ and $m' + \Delta'(\pi) = -1$

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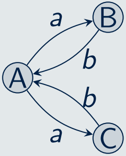
$NFA \subseteq OCN$

Reduction to Trace Universality of OCN

$$\begin{array}{ccc} \text{NFA} & \subseteq & \text{OCN} \\ & \rightsquigarrow & \\ \Sigma^* & \subseteq & \text{OCN} \end{array}$$

OCN Universality: Decidability

Intuition: witnessing non-Universality in a NFA



$$\begin{pmatrix}\top\\ \bot\\ \bot\end{pmatrix} \xrightarrow{a} \begin{pmatrix}\bot\\ \top\\ \top\end{pmatrix} \xrightarrow{b} \begin{pmatrix}\top\\ \bot\\ \bot\end{pmatrix} \xrightarrow{?}{}^{*} \begin{pmatrix}\bot\\ \bot\\ \bot\end{pmatrix}$$

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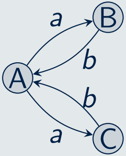
Intuition: witnessing non-Universality in a NFA



$$\begin{pmatrix} \top \\ \bot \\ \bot \end{pmatrix} \xrightarrow{a} \begin{pmatrix} \bot \\ \top \\ \top \end{pmatrix} \xrightarrow{b} \begin{pmatrix} \top \\ \bot \\ \bot \end{pmatrix} \xrightarrow{?}{}^{*} \begin{pmatrix} \bot \\ \bot \\ \bot \end{pmatrix}$$

Observation due to $pm \subseteq p(m+1)$:

Combined traces of *sets* of configurations are representable by maximal elements.

OCN Universality: Decidability

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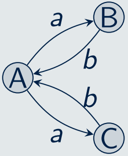

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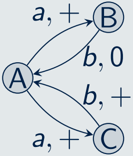
$\rightsquigarrow$ Reachability of $(\bot)^k$ in a “maximizing” k-counter automaton

OCN Universality: Decidability

Intuition: witnessing non-Universality in a OCN



$$\begin{pmatrix} 0 \\ \bot \\ \bot \end{pmatrix} \xrightarrow{a} \begin{pmatrix} \bot \\ 1 \\ 1 \end{pmatrix} \xrightarrow{b} \begin{pmatrix} 2 \\ \bot \\ \bot \end{pmatrix} \xrightarrow{?}{}^{*} \begin{pmatrix} \bot \\ \bot \\ \bot \end{pmatrix}$$

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Fast-Growing Functions $F_n : \mathbb{N} \to \mathbb{N}$

$$F_0(x) = x + 1 \qquad F_{k+1}(x) = F_k^{x+1}(x) \qquad F_\omega(x) = F_x(x).$$

The *Fast-Growing Hierarchy* at level k is the class $\mathfrak{F}_k$ that contains all constants and is closed under substitution, sum, projections, limited recursion and applications of functions F_n for $n \leq k$.

- $\mathfrak{F}_k \approx \mathit{NSPACE}(F_k(1))$, for $k \geq 2$.
- A function is called *Ackermannian* if it is in $\mathfrak{F}_\omega \setminus \bigcup_{k \in \mathbb{N}} \mathfrak{F}_k$.

Theorem

OCN Trace Universality is Ackermannian

in $\mathfrak{F}_\omega$:

naive search for witness as above...

(shortest witnesses are bad *succ*-controlled sequences in $\mathbb{N}_\perp^k$).

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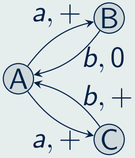
not in $\bigcup_{k\in\mathbb{N}} \mathfrak{F}_k$:

by reduction from the (Ackermannian) control-state reachability problem for lossy counter systems.

OCN Universality: Hardness

Example



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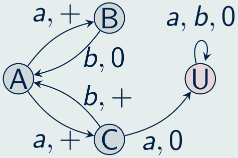
OCN Universality: Hardness

Example: Obstacles



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OCN Universality: Hardness

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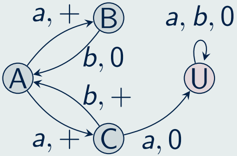

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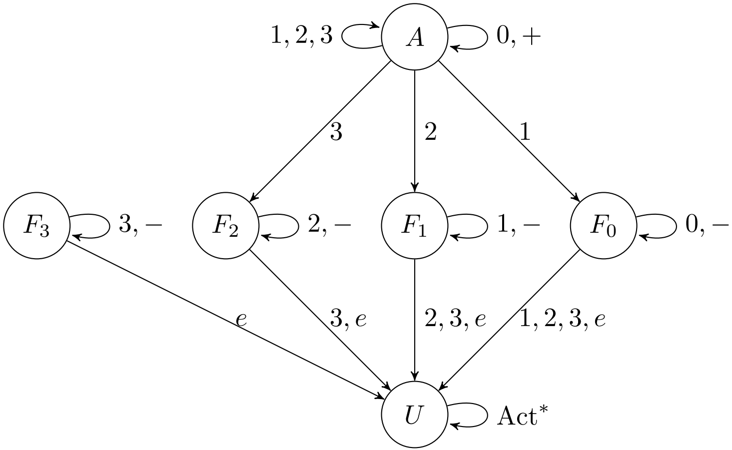
State C is an *obstacle* for letter a:

If $w \in \mathrm{Act}^*$ leads to vector with $v(C) \neq \bot$, then no continuation of wa can be a witness!

Witnesses for non-Universality of length $F_3(0)$



start in $\{A0, F_3 1\}$

Trace Inclusion for One-Counter Automata / Nets

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$\not\subseteq$	NFA	OCN	OCA
NFA	PSPACE	decidable	undecidable
OCN			undecidable
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DFA	NL		
DOCN			
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Questions?

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DOCN	NL	NL	?
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