Markov Decision Processes with Energy-Parity Objectives

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Edinburgh/Liverpool, UK

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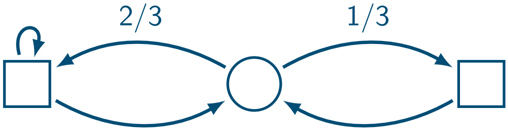
MDPs

Finite graphs, partitioned into *controlled* □ and *random* ○ states;
A prob. dist. over successors for every random state.



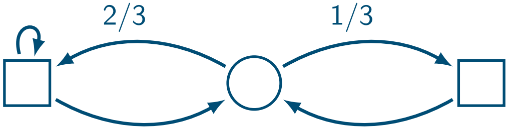
Controller *Strategies*

resolve choice of successor for controlled states to induce a Markov Chain with associated probability space over infinite runs.

The Almost-sure Problem

Does there exist a strategy with $\mathbb{P}^{\sigma}(\mathit{Obj}) = 1$?

Some Objective Functions



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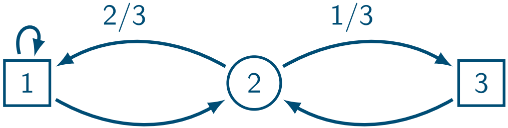

PARITY
maximal colour visited infinitely often is even.

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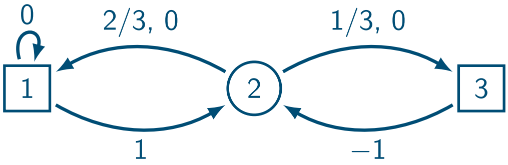

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ENERGY

$\forall n \in \mathbb{N}.\ \sum_{i=0}^{n} \text{cost}(e_i) \geq 0$

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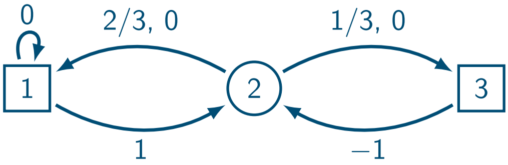

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Positive Mean Payoff
$\lim_{n\to\infty} \sum_{i=0}^{n} \mathrm{cost}(e_i)/n > 0$

Almost-sure Problems for finite MDPs

ENERGY

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- FD determined
- NP $\cap$ coNP
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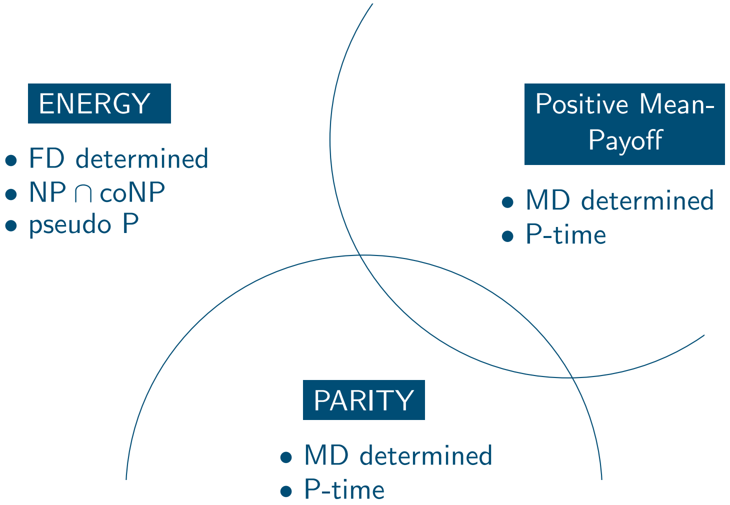

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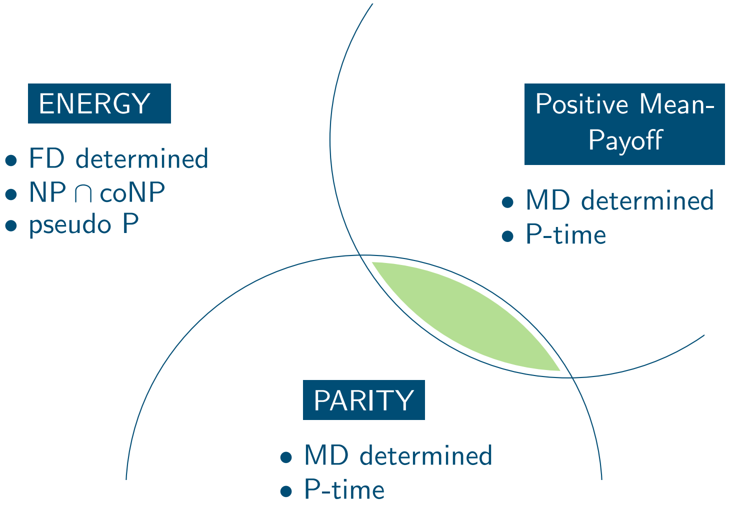

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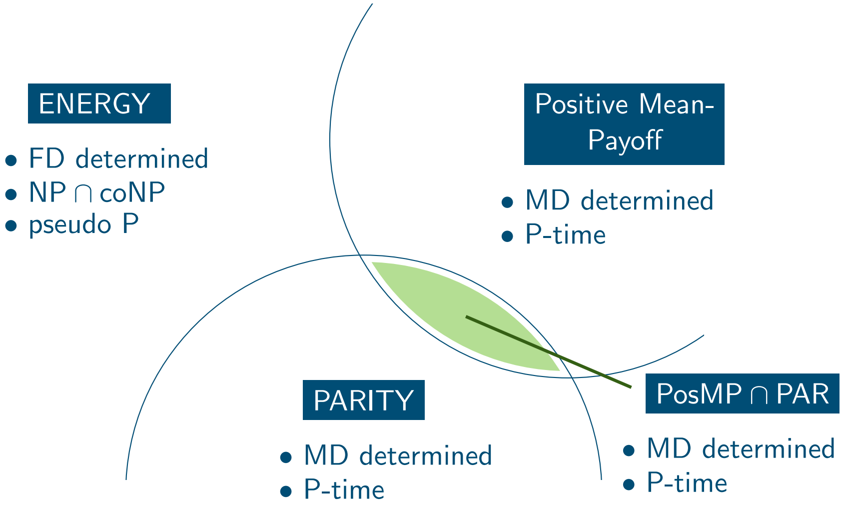

Chatterjee/Doyen, MFCS'11

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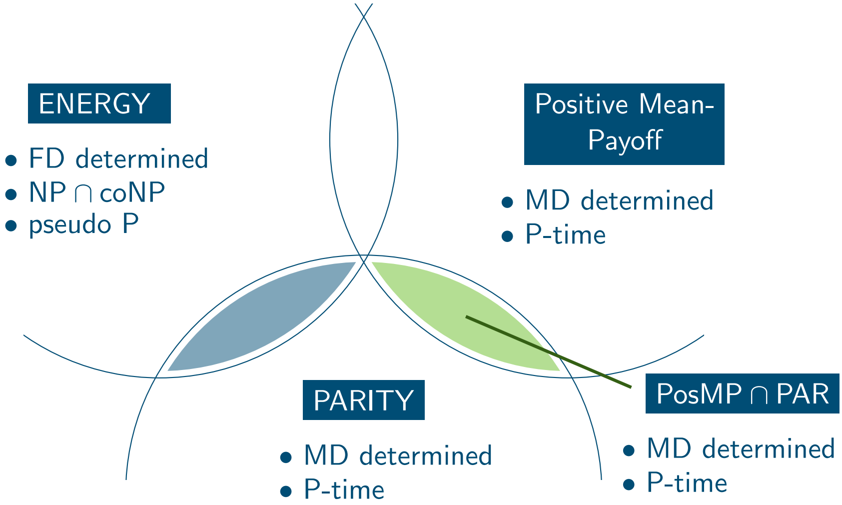

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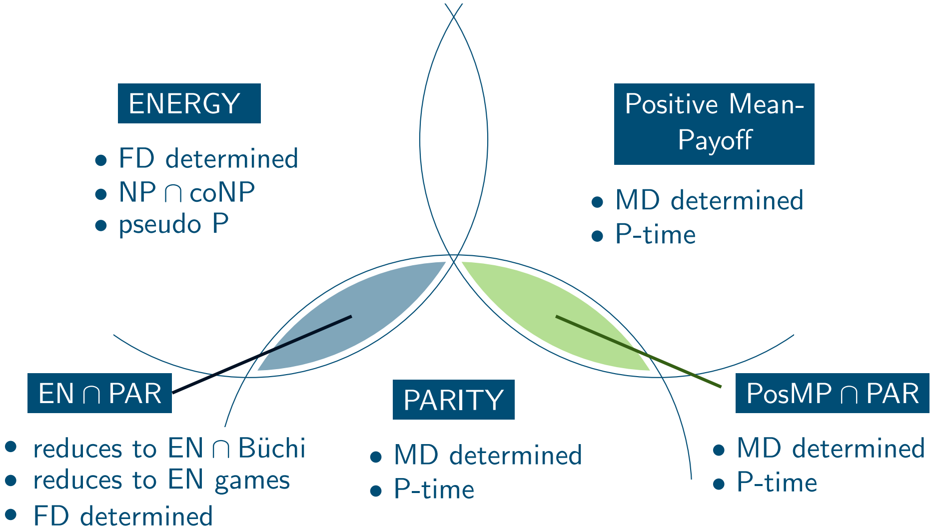

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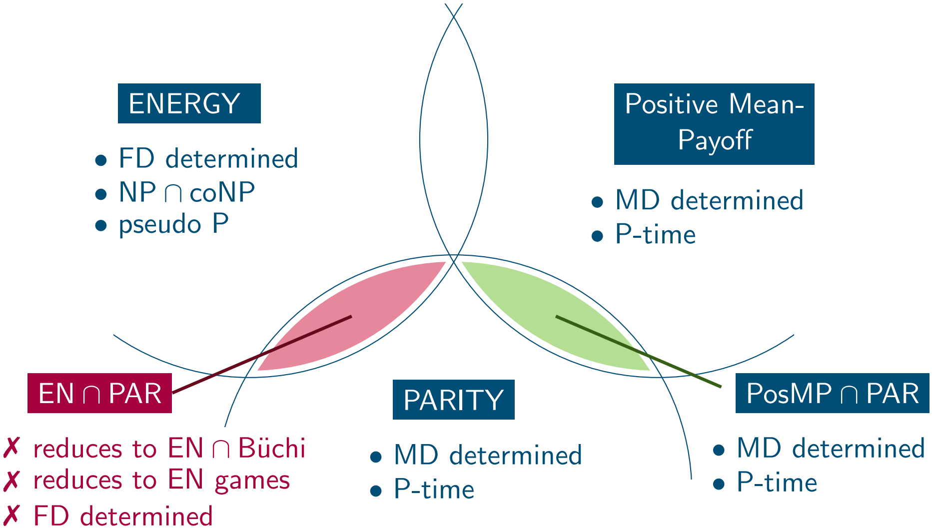

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3. Same bounds hold for the limit-sure problem;

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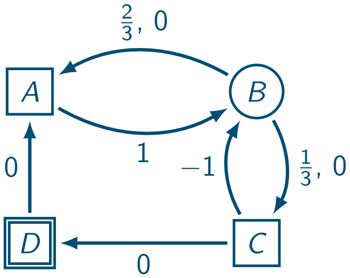

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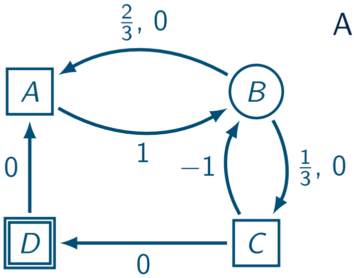

Aim: Satisfy ENERGY and avoid D

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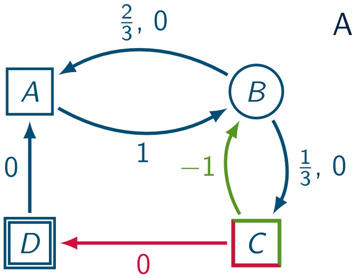

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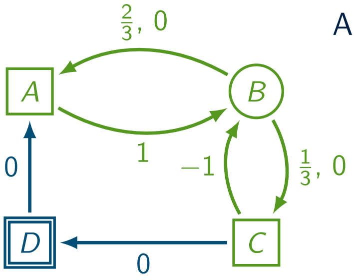

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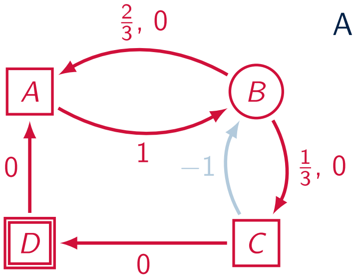

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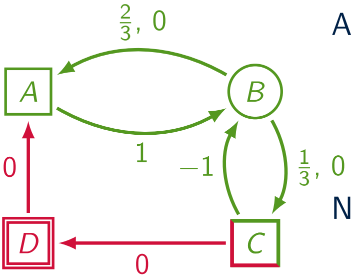

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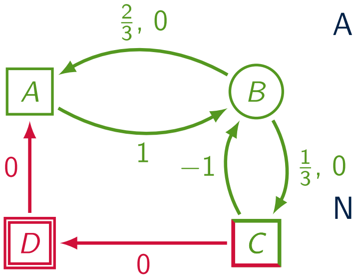

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An (a.s.) winning strategy

- "move to D only if energy level is 0"
- works because $\mathbb{P}^{green}(\text{always} > 0) > 1/2$

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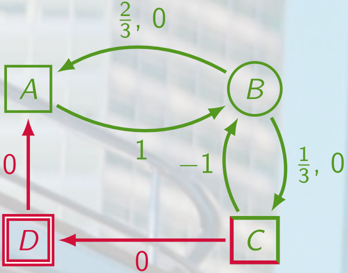
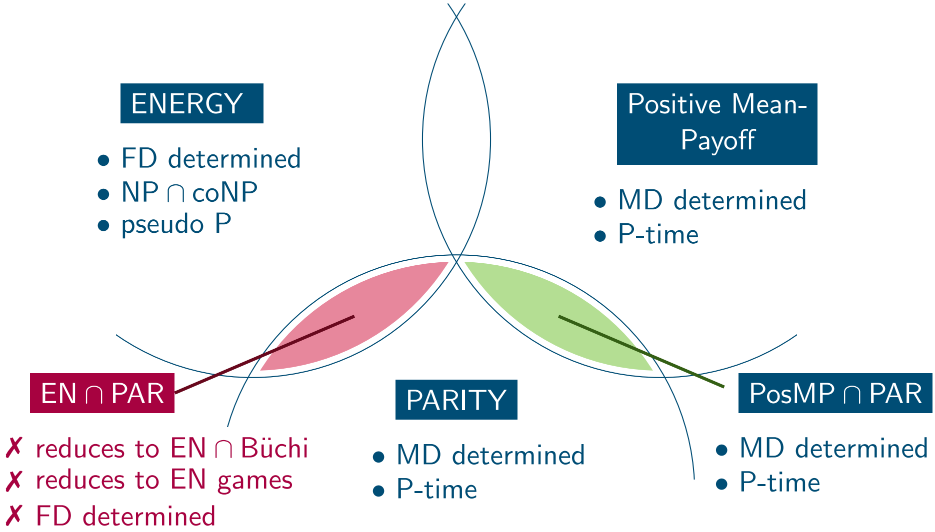


Almost-sure Problems for finite MDPs



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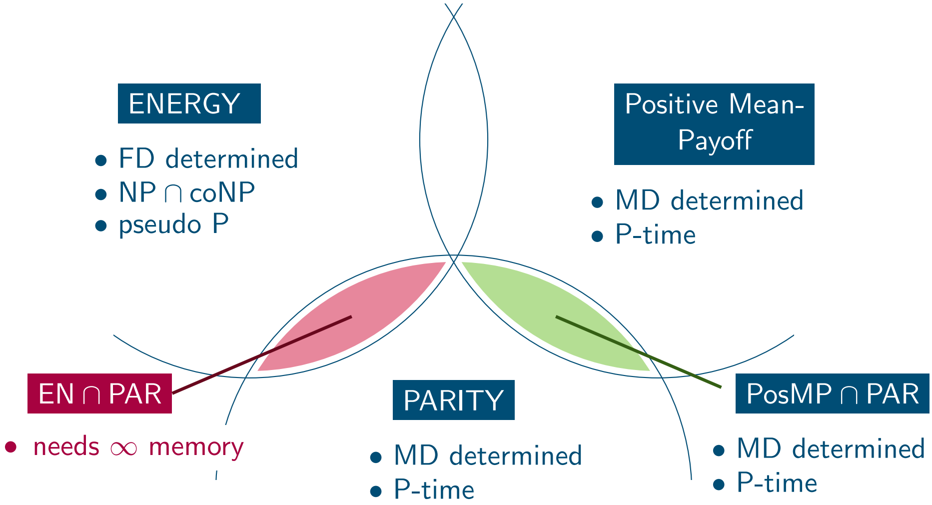

The Storage Objective

A path satiesfies the k-Storage condition $\mathrm{ST}(k)$ if

$$k + \sum_{i=n}^{m} \mathrm{cost}(e_i) \geq 0$$

for all indices $n < m$.

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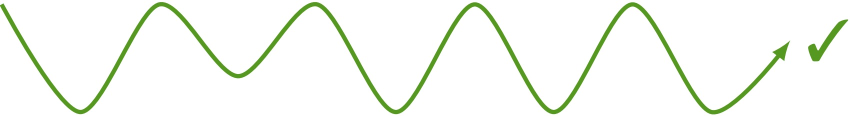

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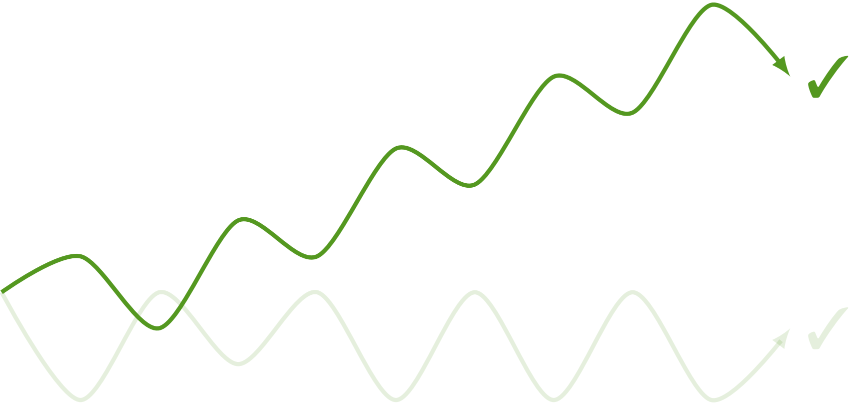

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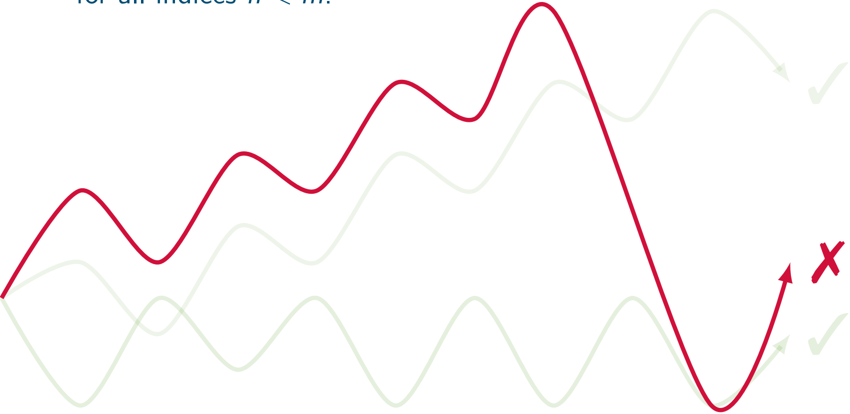

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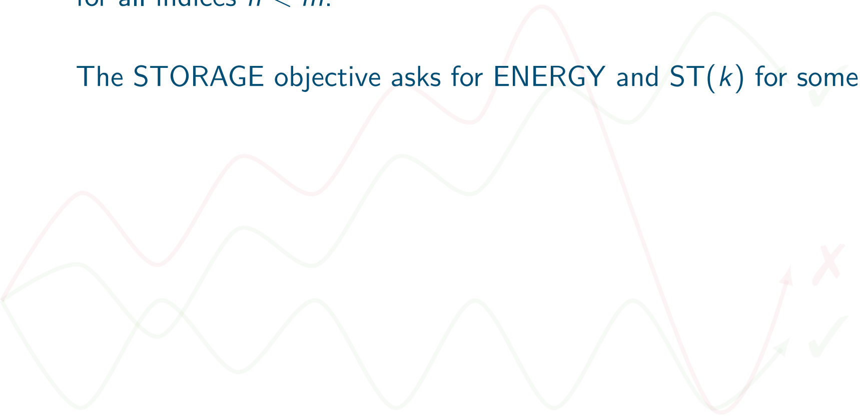

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CAUTION

$$\text{(a.s.) ENERGY} \iff \text{(a.s.) STORAGE}$$

but

$$\text{(a.s.) ENERGY} \cap \text{PARITY} \not\Longleftrightarrow \text{(a.s.) STORAGE} \cap \text{PARITY}$$

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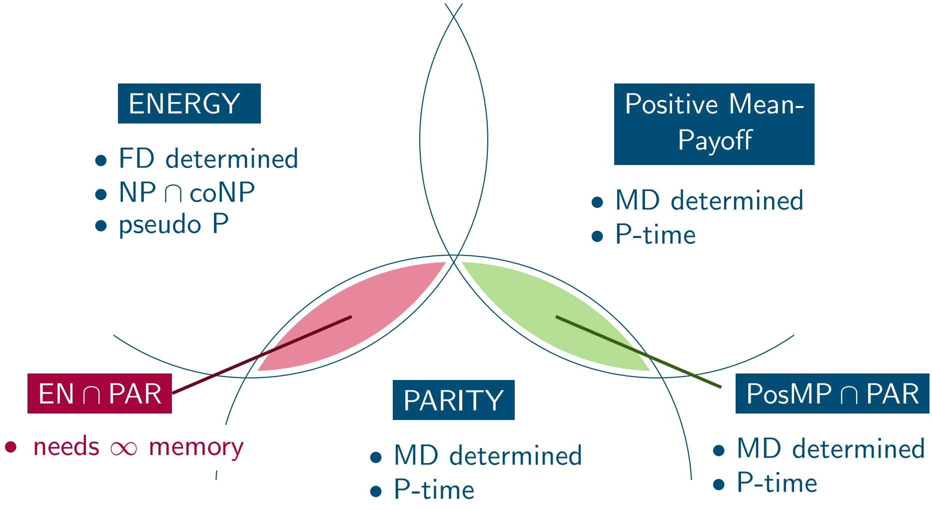

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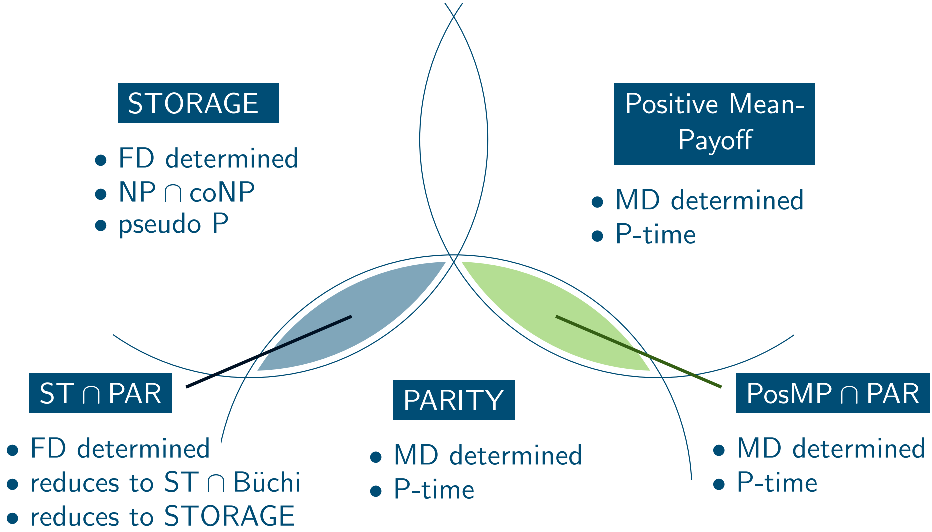

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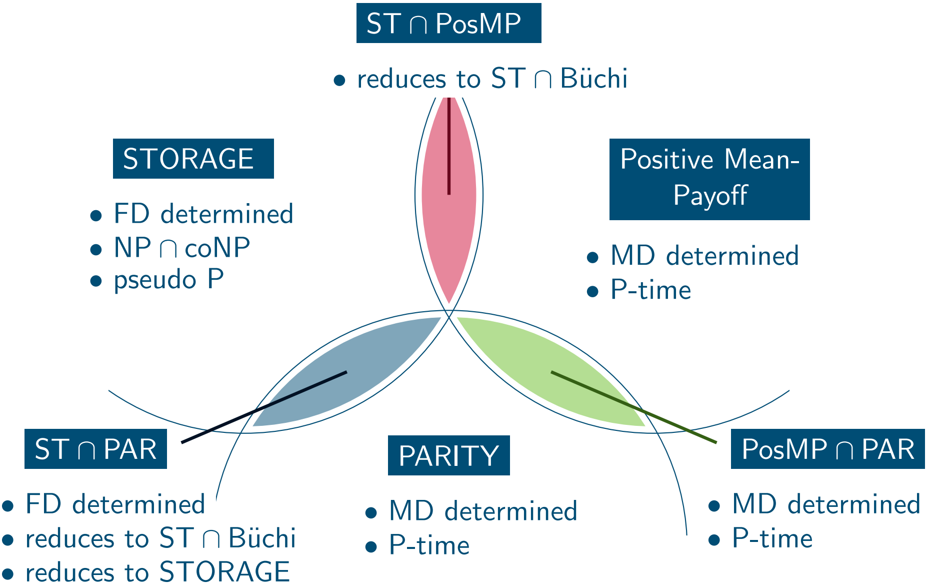

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Computing Winning Sets

Bailout:
arbitrary increase

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$\delta > 0$ chance of success

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switch and try again

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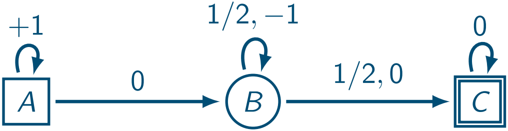

Here, EN $\cap$ Büchi(C) holds limit-surely but not almost-surely in A.

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What next?

- generalize to $2\frac{1}{2}$ player games?
- multiple dimensions?

thank you.