

COMP108 Algorithmic Foundations  
Tutorial 2  
w/c 10th February 2014

Name: \_\_\_\_\_

*Hand this in to the demonstrator at the end of the tutorial even if you haven't finished it. You will get feedback in the next tutorial. Tutorial participation contributes to 5% of overall marks.*

1. Alice goes to the sports centre every  $x$  days on day  $x, 2x, 3x, \dots$  while Bob goes every  $y$  days on day  $y, 2y, 3y, \dots$ . They want to know on which days they will see each other in the sports centre. Write a pseudo code of a while-loop to output all those days for up to 100 days.

For example, if  $x = 4$  and  $y = 6$ , then the output should be 12, 24, 36, 48, 60, 72, 84, 96.

2. Prove by mathematical induction that  $2^0 + 2^1 + 2^2 + \dots + 2^n = 2^{n+1} - 1$ , for all integers  $n \geq 0$ .

3. Prove by mathematical induction about the sum of odd integers:

$$1 + 3 + 5 + \cdots + (2n - 1) = n^2, \text{ for all integers } n \geq 1.$$

4. **[Puzzle]** Nine balls look identical except one is of different weight (can be heavier or lighter). How can you weigh only three times on a balance scale to find out which one is different and whether it is heavier and lighter?



5. **[For those who want more exercises on Mathematical Induction.]**

Prove the following property by mathematical induction.

$$\sum_{i=0}^n (1 + 4i) = (n + 1)(2n + 1) \text{ for all integers } n \geq 0.$$

Note that  $\sum_{i=0}^n (1 + 4i) = 1 + 5 + 9 + \cdots + (1 + 4(n - 1)) + (1 + 4n)$ .