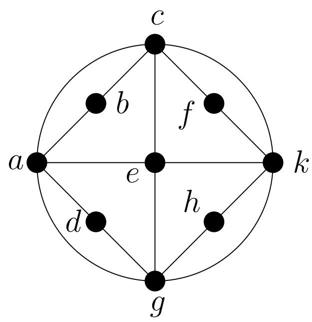
COMP108 Algorithmic Foundations
Tutorial 7
w/c 17th March 2014

Name: ______________________________________

Hand this in to the demonstrator at the end of the tutorial even if you haven't finished it. You will get feedback in the next tutorial. Tutorial participation contributes to 5% of overall marks.

1. Consider the undirected graph G below.



   (a) List all the vertices adjacent to vertex k.

   (b) What is the degree of the vertex d?

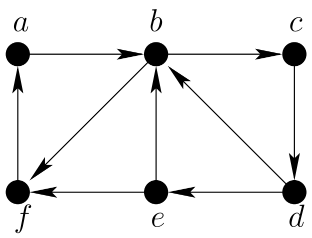
   (c) What is the degree of the graph G (i.e., maximum degree of the vertices)?

   (d) Give the adjacency matrix of the graph G.

   (e) State the conditions for a graph to contain an Euler circuit.

   (f) Does G contain an Euler circuit? If yes, write down the sequence of the vertices in one of these Euler circuits; if no, explain why and suggest the minimum number of edges needed to add to the graph so that an Euler circuit exists.

2. Consider the directed graph G below.

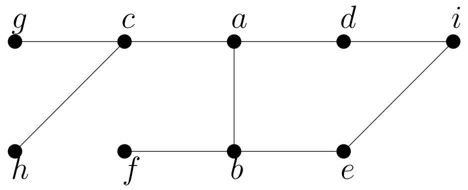


   (a) What is the in-degree and out-degree of the vertex d?

   (b) Give the adjacency matrix of the graph G.

   (c) What is the relationship between the sum of in-degree, the sum of out-degree over all vertices, and the number of edges in a directed graph?

3. Consider the undirected graph G below.



   (a) Starting from the vertex a, write down the vertices in the order of a breadth first search (BFS) traversal.

   (b) Starting from the vertex a, write down the vertices in the order of a depth first search (DFS) traversal.

4. Show that the following recurrence $T(n)$ is $O(\log n)$ using the **substitution method**.

$$T(n) = \begin{cases} 0 & \text{if } n = 1 \\ T(\frac{n}{2}) + 2 & \text{if } n > 1 \end{cases}$$

(Hint: Prove that $T(n) \leq 2 \times \log n$ for all $n \geq 1$.
You can use the fact $\log(\frac{n}{2}) = \log n - \log 2 = \log n - 1$.)

Base case:

When $\boldsymbol{n = 1}$, L.H.S. = ________________________ R.H.S. = ________________________

Therefore, L.H.S. $\leq$ R.H.S.

Induction hypothesis: Assume the property holds for all $n' < n$, in particular, assume that

$$T\left(\frac{n}{2}\right) \leq 2 \times \log\left(\frac{n}{2}\right)$$

Induction step: For n, we want to prove $T(n) \leq 2 \times \log n$.

L.H.S. $= T(n)$	$=$		$\leftarrow$ use the recurrence
	$\leq$		$\leftarrow$ use induction hypothesis
	$=$		$\leftarrow$ simplification from here

Therefore, L.H.S. $\leq$ R.H.S. and the property holds for n.

Conclusion: By the principle of mathematical induction, the property holds for all integers $n \geq 1$.

5. Given a length-n sequence T of characters $T[1..n]$, a length-m sequence of characters $P[1..m]$ is called a *substring* of T if there exists some $1 \le i \le n - m + 1$ such that $T[i..(i + m-1)]$ is the same as $P[1..m]$.
   For example, if T is ACGTACGGG, then ACGG is a substring appearing once in T, ACG is a substring appearing twice and ACC is not a substring at all.

   Design and write a pseudo code algorithm to count how many times P appears as a substring of T.

   What is the worst case time complexity of your algorithm (in big-O notation)? Explain briefly.

6. **[Puzzle]** Suppose there are 10 people in a room. Each person shakes hands with some other people in the room. Prove that the number of people having an odd number of handshakes is even.