Uniform Argumentation Frameworks

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Abstract. We introduce a derivative of Dung's seminal abstract argumentation frameworks (AFs) through which distinctive features both of Dung's semantics and so-called "*value-based*" argumentation frameworks (VAFs) may be captured. These frameworks, which we describe as *uniform* AFs, thereby recognise that, in some circumstances, arguments may be deemed acceptable, not only as a consequence of subjective viewpoints (as are modelled by the concept of audience in VAFs) but also as a consequence of "value independent" acceptance of other arguments: for example in the case of factual statements. We analyse divers acceptability conditions for arguments in uniform AFs and obtain a complete picture for the computational complexity of the associated decision questions. Amongst other results it is shown that reasoning in uniform AFs may pose significantly greater computational challenges than either standard or value-based questions, a number of problems being complete for the third level of the polynomial hierarchy.

Keywords. abstract argumentation frameworks, practical reasoning, computational complexity

1. Introduction

When thinking about human reasoning, one important consideration is the direction of fit, a distinction presented by Anscombe in [2]. Broadly this distinguishes reasoning about what is that case, in which our *beliefs* are intended to fit the world as it is, from reasoning about what should be done, in which we choose an action which will fit the world to our *desires*. This can serve to differentiate between theoretical reasoning, which aims at *discovering* what *is* true, from practical reasoning which is directed towards *choosing* what *should be* true. The distinction is reflected in work on abstract argumentation by the treatment of the attack relation. In the original conception of argumentation frameworks [10], attack and defeat were synonymous: the only way an argument could resist an attacker was to find an argument which would defeat that attacker. This is appropriate for reasoning about what is the case, where the fit is from the world. This is not, however, sufficient for domains where choice is important and the direction of fit is to the world, such as politics, law and practical reasoning generally. In such domains, an undefeated attacker can be resisted on the basis of the preferences and aspirations of the *audience* [15]. To accommodate such reasoning, argumentation frameworks that distinguish attack from defeat for a particular audience have been developed, including Preference-Based Argumentation Frameworks [1], and

Value-based Argumentation Frameworks (VAFs) [6]. Whereas in standard frameworks we can speak of credulous and sceptical acceptance to distinguish what can be accepted from what must be accepted, for VAFs it is appropriate to refer to objective and subjective acceptability (acceptable to *all* or *some* audiences respectively).

Although for individual arguments the direction of fit is clear, many actual reasoning situations require consideration of a number of arguments, some with one direction of fit and some with the other. Normally, when choosing what to do we are not free to choose whatever we desire, but rather find that our desires are constrained by what is true in the current situation. If choosing how to travel to Vienna, that there are flights only from certain airports restricts the choice of airport. Sometimes too, my desires can constrain what can be the case: a person who will not, from fear of flying, travel by air, cannot be in Vienna in twenty four hours time. Currently this need to constrain choice by what is the case is often handled (e.g. [5]) by dividing the process of decision making into stages, and assuming the factual issues to be resolved before considering matters of choice, or in VAFs by making 'truth' the most preferred value. The latter approach, however, inappropriately blurs the distinction between directions of fit, and so softens the constraint of truth. Moreover, sometimes, division into stages is not possible, and it is necessary to consider arguments with both directions of fit *together*. We give some examples of domains where this is needed in Section 5.

Earlier approaches to this issue can be found in [5] which considers theoretical reasoning and practical reasoning in separate stages and [13], which uses sceptical semantics for theoretical reasoning and credulous semantics for practical reasoning. It should be noted that, although superficially similar, the mechanisms of [14] and its so-called "multi-sorted frameworks" are unrelated to our work: [14] being concerned – in their phrasing – with "how to define an abstract argumentation where the arguments can be evaluated under different semantics". In particular, the "partitioning" approach from [14] does not consider VAF semantics or the interaction between "cells" of a partition in the sense arising in the decision problems addressed in the present paper.

In this paper, to allow both kinds of argument to be considered in a single, uniform, framework and to illuminate some important distinctions relating to how they are considered, we will consider argumentation frameworks that include both standard arguments and arguments associated with values, which we will refer to these as *Uniform Argumentation Frameworks* (UAFs).

One domain in which there is a need for both kinds of framework to be considered simultaneously is law. As discussed in [4], the resolution of legal cases requires both judgement as to the facts of the case, and decisions as to the interpretation of the law. While the latter can be seen as choices made in the light of the social purposes that the law is intended to serve, the former must attempt to fit what was in fact the case. Here the choices are constrained by facts. If we turn our attention to the US Supreme Court, however, we see that what can be accepted as legally true is constrained by the choices enshrined in the constitution. Although currently in the USA the existence of capital punishment is credulously acceptable (and is indeed true in some states but not others) some interpretations of the Eighth Amendment would make this otherwise, so that such a sanction is subjectively sceptically unacceptable (indeed that this position is so under the current constitution has been argued by some Justices, e.g. Marshall in *Furman v Georgia* (408 U.S. 238 (1972)), albeit unsuccessfully).

We give the necessary background definitions in Section 2, and the results and proofs giving a complete characterization of the complexity of UAFs for preferred semantics in Section 4. We also note some points of interest concerning these complexity results, and make some observations about the complexity of UAFs under other semantics. In Section 5 we discuss the implications of these results for reasoning problems in particular contexts, and Section 6 offers a brief conclusion.

2. Background Definitions

We begin by recalling the concept of abstract argumentation framework and terminology from [10] and outline the main computational problems that have been of interest within this framework.

Definition 1 *An* argumentation framework *(*AF*) is a pair $\mathcal{H} = \langle \mathcal{X}, \mathcal{A} \rangle$, in which $\mathcal{X}$ is a finite set of* arguments *and $\mathcal{A} \subseteq \mathcal{X} \times \mathcal{X}$ is the* attack relationship *for $\mathcal{H}$. A pair $\langle x, y \rangle \in \mathcal{A}$ is referred to as 'y* is attacked by *x' or 'x* attacks *y'.*

For $S \subseteq \mathcal{X}$,

$$\begin{array}{lll} S^- & =_{\text{def}} & \{\, p \;:\; \exists\, q \in S \;\; \textit{such that} \; \langle p, q \rangle \in \mathcal{A} \} \\ S^+ & =_{\text{def}} & \{\, p \;:\; \exists\, q \in S \;\; \textit{such that} \; \langle q, p \rangle \in \mathcal{A} \} \end{array}$$

When S is some subset of $\mathcal{X}$, we use $\mathcal{H}_{ind(S)}$ to denote the AF *with arguments S and attacks $\mathcal{A} \cap (S \times S)$.*

An argument $x \in \mathcal{X}$ is acceptable with respect to S *if for every $y \in \mathcal{X}$ that attacks x there is some $z \in S$ that attacks y. A subset, S, is* conflict-free *if no argument in S is attacked by any other argument in S. A conflict-free set S is* admissible *if every $y \in S$ is acceptable w.r.t S and S is a* preferred extension *if it is a maximal (with respect to $\subseteq$) admissible set. A subset, S, is a* stable extension *if S is conflict free and every $y \notin S$ is attacked by S. From [8], S is a* semi-stable extension *if S is admissible and for any set T should $S \cup S^+ \subset T \cup T^+$ then T is* not *admissible. The* grounded extension *of $\langle \mathcal{X}, \mathcal{A} \rangle$ is the subset $\mathcal{X}$ obtained by iterating the following process: given $S \subseteq \mathcal{X}$, let $\mathcal{F}(S)$ be the set of arguments* acceptable *to S. Letting $\mathcal{F}^0(S)$ denote S and $\mathcal{F}^{i+1}(S) = \mathcal{F}(\mathcal{F}^i(S))$ ($i \geq 0$), the* grounded extension *of $\langle \mathcal{X}, \mathcal{A} \rangle$ is the least fixed point of $\mathcal{F}(\emptyset)$, i.e. the set of arguments $\mathcal{F}^k(\emptyset)$ where k is the smallest value satisfying $\mathcal{F}^k(\emptyset) = \mathcal{F}^{k+1}(\emptyset)$. [10] shows that the grounded extension is well-defined and unique.*

We use σ to denote one of the (extension) type semantics {ADM,PR,ST,GR,SST} corresponding to admissible sets, preferred, stable, grounded and semi-stable extensions.

For a given semantics σ and AF, $\mathcal{H}(\mathcal{X}, \mathcal{A})$ we use $\mathcal{E}_\sigma$ to denote the set of all subsets of $\mathcal{X}$ that satisfy the conditions specified by σ, and in the event of σ being a *unique status* semantics (i.e. for all $\mathcal{H}(\mathcal{X}, \mathcal{A})$, $|\mathcal{E}_\sigma(\mathcal{H})| = 1$) we denote the unique extension by $E_\sigma(\mathcal{G})$.

Table 1. Decision Problems in AFs

Problem Name	Question
Verification (VER_σ)	Is $S \in \mathcal{E}_\sigma(\mathcal{H})$?
Credulous Acceptance (CA_σ)	$\exists\, S \in \mathcal{E}_\sigma(\mathcal{H})$ for which $x \in S$?
Sceptical Acceptance (SA_σ)	$\forall\, T \in \mathcal{E}_\sigma(\mathcal{H})$ is $x \in T$?
Existence (EXISTS_σ)	Is $\mathcal{E}_\sigma(\mathcal{H}) \neq \emptyset$?
Emptiness ($\text{VER}^{\emptyset}_\sigma$)	Is $\mathcal{E}_s(\mathcal{H}) = \{\emptyset\}$?

Informally, the canonical decision problems are *Verification* (VER), *Credulous Acceptance* (CA) and *Sceptical Acceptance* (SA): VER_σ, refers to the decision problem of verifying that a given set of arguments satisfies the conditions of the semantics σ, i.e. that the set is in the collection $\mathcal{E}_\sigma$; CA_σ that of deciding if a given argument, x, is a member of some set, S, in $\mathcal{E}_\sigma$; while SA_σ asks whether an argument belongs to *every* set in $\mathcal{E}_\sigma$. The formal definitions of these problems for AFs is presented in Table 1. In the case of preferred extensions we note that $\text{SA}_{pr}(\mathcal{H}, x)$ is captured by the quantified formula:

$$\forall\, S \subseteq \mathcal{X} \exists\, T \subseteq \mathcal{X}\ (x \in S)\ \vee\ (S \notin \mathcal{E}_{adm}(\mathcal{H})) \vee \\ ((S \subset T) \wedge (T \in \mathcal{E}_{adm}(\mathcal{H}))$$

whose satisfiability can be decided in Π_2^p: i.e. x is sceptically accepted wrt preferred extensions iff for any subset S of $\mathcal{X}$, either x in a member of S or S fails to be a *maximal* admissible set.

In [6], Bench-Capon introduced *value-based* argumentation frameworks (VAFs), which provide a mechanism for describing the phenomenon that the acceptability status of an argument may be coloured by the fact that its endorsers view the value (in the sense of ethical, legal or other qualitative assessment) as having greater importance than the values promoted by the argument's attackers.

Definition 2 *A* value-based argumentation framework *(subsequently,* VAF*) is defined by a tuple $\mathcal{H} = \langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle$ in which the pair $\langle \mathcal{X}, \mathcal{A} \rangle$ forms a standard* AF *(in the sense of Defn. 1), $\mathcal{V} = \{v_1, v_2, \ldots, v_k\}$ is a set of* values *and $\eta\ :\ \mathcal{X} \to \mathcal{V}$ a mapping which associates a value in $\mathcal{V}$ with each $x \in \mathcal{X}$. A specific audience over $\mathcal{V}$ is a total ordering, $\succ$, of $\mathcal{V}$. For such an audience, α, an attack $\langle x, y \rangle \in \mathcal{A}$ is said to be* successful *if it is* not *the case that $\eta(y) \succ_\alpha \eta(x)$, i.e. when x and y have the same value then $\langle x, y \rangle$ is always successful otherwise $\langle x, y \rangle$ succeeds with respect to α only if $\eta(x) \succ_\alpha \eta(y)$: the value promoted by x is considered more important (to the audience α) than that supported by y.*

For a specific audience α and VAF *$\mathcal{H}(\langle \mathcal{X}, \mathcal{A}, \mathcal{V}, \eta \rangle)$ the standard* AF induced by *α, $\mathcal{H}^{(\alpha)}$, has arguments $\mathcal{X}$ and attack set $\mathcal{A}_\alpha$ given by*

$$\mathcal{A}_\alpha \quad = \quad \mathcal{A} \setminus \{\langle x, y \rangle\ :\ \eta(y) \succ_\alpha \eta(x)\}$$

so that $\mathcal{A}_\alpha$ contains only those attacks in $\mathcal{A}$ which are successful w.r.t. α.

The concept of induced framework now allows the set of subsets, $\mathcal{E}_{pr}^{vaf}$ to be described through,

$$\mathcal{E}_{pr}^{vaf}(\mathcal{H}) \quad = \quad \bigcup_{\alpha} \mathcal{E}_{pr}(\mathcal{H}^{(\alpha)})$$

In Bench-Capon's original presentation the restriction that VAF*s do not contain* directed *cycles of arguments with identical values is imposed. Although this suffices to ensure that $\mathcal{E}_{pr}(\mathcal{H}^{(\alpha)})$ contains exactly one set (since the induced* AF *is acyclic), even with this restriction present, $\mathcal{E}_{pr}^{vaf}$ as defined fails to be an* extension-based *semantics, i.e. one may easily construct* VAF*s, $\mathcal{H}$, having audiences, α and β for which $\mathcal{E}_{pr}(\mathcal{H}^{(\alpha)}) = \{S_\alpha\}$, $\mathcal{E}_{pr}(\mathcal{H}^{(\beta)}) = \{S_\beta\}$ and $S_\alpha \subset S_\beta$.*

The decision problems subjective *(*SBA*) and* objective *(*OBA*) acceptance with instances of a* VAF*, $\mathcal{H}$, and argument x, are given by,*

$$\begin{array}{lcl} \text{SBA}(\langle \mathcal{H}, x\rangle) & \Leftrightarrow & \exists\ \alpha\ s.t.\ \text{CA}_{pr}(\langle \mathcal{H}^{(\alpha)}, x\rangle) \\ \text{OBA}(\langle \mathcal{H}, x\rangle) & \Leftrightarrow & \forall\ \alpha\ \text{CA}_{pr}(\langle \mathcal{H}^{(\alpha)}, x\rangle) \end{array}$$

3. Uniform Argumentation Frameworks

We may now introduce the main innovation of the paper: *uniform* argumentation frameworks. The basic idea underlying the concept of a *uniform* AF is that of combining standard abstract AFs, as described in Defn. 1 with the less abstracted approach offered in VAFs. Formally,

Definition 3 *A* uniform argumentation framework *(*UAF*) is described by a quintuple $\mathcal{M} = \langle \mathcal{Y}, \mathcal{Z}, \mathcal{A}, \mathcal{V}, \eta\rangle$ in which $\langle \mathcal{Y} \cup \mathcal{Z}, \mathcal{A}\rangle$ defines a standard* AF *and $\langle \mathcal{Z}, \mathcal{A} \cap \mathcal{Z} \times \mathcal{Z}, \mathcal{V}, \eta\rangle$ describes a* VAF *(note that $\mathcal{Y}$ and $\mathcal{Z}$ are disjoint sets and the value mapping η only applies to $\mathcal{Z}$*[1]*).*

Given a specific audience, α, the induced AF*, $\mathcal{M}^{(\alpha)}$, has arguments $\mathcal{X} = \mathcal{Y} \cup \mathcal{Z}$ and attacks*

$$\mathcal{A}_\alpha \quad = \quad \mathcal{A} \setminus \{\ \langle v, w\rangle\ :\ v \in \mathcal{Z},\ w \in \mathcal{Z}\ \textit{and}\ \eta(w) \succ_\alpha \eta(v)\ \}$$

It should be noted that we retain the requirement that the VAF $\langle \mathcal{Z}, \mathcal{A} \cap \mathcal{Z} \times \mathcal{Z}, \mathcal{V}, \eta\rangle$ satisfies the property that every directed cycle within it involves at least two distinct values, observing that if $\mathcal{C} \subseteq \mathcal{Z}$ are a set of like-valued arguments linked in a directed cycle, then it may be more reasonable to consider $\mathcal{C}$, not as value-based *per se* but rather as arguments that should be considered within $\mathcal{Y}$.

Definition 4 *Let $\mathcal{M} = \langle \mathcal{Y}, \mathcal{Z}, \mathcal{A}, \mathcal{V}, \eta\rangle$ be a* UAF *and $x \in \mathcal{Y} \cup \mathcal{Z}$. Table 2 defines the* eight *basic statuses that an argument x may have. In each case the problem instance is a* UAF*, $\mathcal{M}$, together with an argument x; α is a specific audience. The notation $S_\mathcal{Y}$ is used for an arbitrary subset of $\mathcal{Y}$, while $S_\mathcal{Z}$ indicates a subsets of $\mathcal{Z}$; more generally, S is a subset of $\mathcal{X} = \mathcal{Y} \cup \mathcal{Z}$*

We focus, at first, on *preferred* semantics, so that the basis structure with respect to which credulous and sceptical acceptance are defined is $\mathcal{E}_{pr}$.

[1] An alternative would be to fix η as a *partial* function, so that those x with $\eta(x)$ undefined, would form the set $\mathcal{Y}$. For reasons connected with extended developments of the notion of UAF (see Baroni *et al.* [3]) we do not adopt this approach.

Table 2. Argument Status in Uniform AFs: arbitrary σ

Property	Decision Question
subjective-σ-credulous ($\textsc{sba}_{\sigma}^{\textsc{CA}}$)	$\exists\alpha\exists S\ \{x\}\cup S\in\mathcal{E}_{\sigma}(\mathcal{M}^{(\alpha)})$?
subjective-σ-sceptical ($\textsc{sba}_{\sigma}^{\textsc{SA}}$)	$\exists\alpha\forall S\ S\in\mathcal{E}_{\sigma}(\mathcal{M}^{(\alpha)})\Rightarrow x\in S$?
objective-σ-credulous ($\textsc{oba}_{\sigma}^{\textsc{CA}}$)	$\forall\alpha\exists S\ \{x\}\cup S\in\mathcal{E}_{\sigma}(\mathcal{M}^{(\alpha)})$?
objective-σ-sceptical ($\textsc{oba}_{\sigma}^{\textsc{SA}}$)	$\forall\alpha\forall S\ S\in\mathcal{E}_{\sigma}(\mathcal{M}^{(\alpha)})\Rightarrow x\in S$?
σ-credulous-subjective ($\textsc{ca}_{\sigma}^{\textsc{SBA}}$)	$\exists S\exists\alpha\ \{x\}\cup S\in\mathcal{E}_{\sigma}(\mathcal{M}^{(\alpha)})$?
σ-credulous-objective ($\textsc{ca}_{\sigma}^{\textsc{OBA}}$)	$\exists S_{\mathcal{Y}}\forall\alpha\ S_{\mathcal{Y}}\cup E_{pr}(\mathcal{M}^{(\alpha)}_{ind(\mathcal{Z}\setminus S^{+}_{\mathcal{Y}})})\ \cup\ \{x\}\in\mathcal{E}_{\sigma}(\mathcal{M}^{(\alpha)})$?
σ-sceptical-subjective ($\textsc{sa}_{\sigma}^{\textsc{SBA}}$)	$\forall S\exists\alpha\ S\in\mathcal{E}_{\sigma}(\mathcal{M}^{(\alpha)})\Rightarrow x\in S$?
σ-sceptical-objective ($\textsc{sa}_{\sigma}^{\textsc{OBA}}$)	$\forall S\forall\alpha\ S\in\mathcal{E}_{\sigma}(\mathcal{M}^{(\alpha)})\Rightarrow x\in S$?

4. Complexity in Uniform Frameworks

We recall that, for $k\geq 1$, the Σ_k^p–complete (resp. Π_k^p–complete) decision problems $\textsc{qsat}_k^{\exists}$ ($\textsc{qsat}_k^{\forall}$) take as instances CNF formulae, $\varphi(X_1,\ldots,X_k)$ with X_i a set of n propositional variables and $X_i\cap X_j=\emptyset$ whenever $i\neq j$. Such instances are accepted if and only if there is some assignment, $\underline{a}$ of Boolean values to X_1 (resp. for *every* choice of value assignment, $\underline{a}$ to X_1) the CNF formula given by $\varphi(\underline{a},X_2,\ldots,X_k)$ is accepted as an instance of $\textsc{qsat}_{k-1}^{\forall}$ (resp. $\textsc{qsat}_{k-1}^{\exists}$). For $k=1$ the cases $\textsc{qsat}_1^{\exists}$ ($\textsc{qsat}_1^{\forall}$) are usually referred to as CNF *satisfiability* (SAT) (resp. CNF *unsatisfiability* – UNSAT). We first observe that the total number of properties introduced in Defn. 4 is easily reduced from eight to six.

Lemma 1 *For any* UAF, $\mathcal{M}=\langle\mathcal{Y},\mathcal{Z},\mathcal{A},\mathcal{V},\eta\rangle$ *and any argument* $x\in\mathcal{Y}\cup\mathcal{Z}$, *it holds that*

$$\begin{array}{rcl}\textsc{sba}_{pr}^{\textsc{CA}}(\mathcal{M},x) & \Leftrightarrow & \textsc{ca}_{pr}^{\textsc{SBA}}(\mathcal{M},x)\\ \textsc{oba}_{pr}^{\textsc{SA}}(\mathcal{M},x) & \Leftrightarrow & \textsc{sa}_{pr}^{\textsc{OBA}}(\mathcal{M},x)\end{array}$$

Proof: Immediate from the definitions. The first of these concerns arguments that can always be interpreted as acceptable by some appropriate audience, and the second arguments that no audience can ever reject. □

For these six problems, it is straightforward to obtain exact complexity classifications in three cases from existing results on the computational complexity of $\textsc{ca}_{pr}$ and $\textsc{sa}_{pr}$ in Dimopoulos and Torres [9] and Dunne and Bench-Capon [11].

Lemma 2

a. $\textsc{sba}_{pr}^{\textsc{CA}}$ *is* NP*–complete.*
b. $\textsc{oba}_{pr}^{\textsc{SA}}$ *is* Π_2^p*–complete.*
c. $\textsc{sa}_{pr}^{\textsc{SBA}}$ *is* Π_2^p*–complete.*

Proof: The upper bounds in all three cases are obvious from the quantifier patterns in Defn. 4 (noting the formulation of $\textsc{sa}_{pr}(\mathcal{H},x)$ given earlier). For the hardness (lower bound) results: in (a) it suffices to note that $\textsc{ca}_{pr}$ is NP–hard and so simply by considering $\mathcal{Z}=\emptyset$ (or $\mathcal{Z}$ whose size is constant, i.e. independent of $\mathcal{Y}$) one has a trivial polynomial time reduction from $\textsc{ca}_{pr}$ to $\textsc{sba}_{pr}^{\textsc{CA}}$; for (b) and (c) a similar reduction combined with the fact that SA$_{pr}$ is Π_2^p–complete gives the lower bounds. □

As a result of the classifications obtained through Propn. 2, we are left with three decision questions in UAFs lacking an exact complexity classification: SBA$^{\text{SA}}$, OBA$^{\text{CA}}$ and CA$^{\text{OBA}}$. Noting that SBA$^{\text{SA}} \in \Sigma_3^p$, OBA$^{\text{CA}} \in \Pi_2^p$ and CA$^{\text{OBA}} \in \Sigma_2^p$ are, again, immediate from Defn. 4, we now show that, in fact, these bounds are exact.

Theorem 1

a. SBA$_{pr}^{\text{SA}}$ *is* Σ_3^p*–complete.*
b. OBA$_{pr}^{\text{CA}}$ *is* Π_2^p*–complete.*
c. CA$_{pr}^{\text{OBA}}$ *is* Σ_2^p*–complete.*

Proof: As we have noted already, it is only needed to establish the relevant hardness results.

To see that SBA$_{pr}^{\text{SA}}$ is Σ_3^p–hard we reduce from the problem QSAT$_3^{\exists}$. Given $\varphi(U,V,W)$ with clauses $\{C_1, C_2, \ldots, C_m\}$ we construct a UAF, $\mathcal{M}_\varphi = \langle \mathcal{Y}_\varphi, \mathcal{Z}, \varphi, \mathcal{A}_\varphi, \mathcal{V}_\varphi, \eta \rangle$, as follows.

The set $\mathcal{Y}_\varphi$ contains arguments $\{\ v_i,\ \neg v_i,\ w_i,\ \neg w_i\}$ for each $v_i \in V$ and $w_i \in W$. In addition $\mathcal{Y}_\varphi$ contains arguments $\{C_1, \ldots, C_m, \varphi, b_1, b_2, b_3\}$.

The set $\mathcal{Z}_\varphi$ contains arguments $\{u_i,\ \neg u_i\ :\ 1 \le i \le n\}$.

The attack relation, $\mathcal{A}_\varphi$ consists of:

$$
\begin{array}{l}
\{\ \langle u_i, \neg u_i \rangle,\ \langle \neg u_i, u_i \rangle\ :\ 1\ \le i \le n\ \} \cup \\
\{\ \langle v_i, \neg v_i \rangle,\ \langle \neg v_i, v_i \rangle\ :\ 1\ \le i \le n\ \} \cup \\
\{\ \langle w_i, \neg w_i \rangle,\ \langle \neg w_i, w_i \rangle\ :\ 1\ \le i \le n\} \cup \\
\{\ \langle x_i, C_j \rangle\ :\ x_i \in U \cup V \cup W \text{ and } x_i \text{ is in clause } C_j\ \} \cup \\
\{\ \langle \neg x_i, C_j \rangle\ :\ x_i \in U \cup V \cup W \text{ and } \neg x_i \text{ is in clause } C_j\ \} \cup \\
\{\ \langle C_j, \varphi \rangle\ :\ 1 \le j \le m\ \} \cup \\
\{\ \langle \varphi, b_1 \rangle,\ \langle \varphi, b_2 \rangle,\ \langle \varphi, b_3 \rangle,\ \langle b_1, b_2 \rangle,\ \langle b_2, b_3 \rangle,\ \langle b_3, b_1 \rangle\ \} \cup \\
\{\ \langle b_3, w_i \rangle,\ \langle b_3, \neg w_i \rangle\ :\ 1 \le i \le n\}
\end{array}
$$

The set of values, $\mathcal{V}_\varphi$, are $\{pos_i,\ neg_i\ :\ 1 \le i \le n\}$ with the mapping η specified as,

$$\eta(u_i) \ = \ pos_i \quad ; \quad \eta(\neg u_i) \ = \ neg_i$$

The instance of SBA$_{pr}^{\text{SA}}$ is completed by setting the argument of interest to be φ. We now claim that $\varphi(U,V,W)$ is accepted as an instance of QSAT$_3^{\exists}$ if and only if $\langle \mathcal{M}_\varphi, \varphi \rangle$ is accepted as an instance of SBA$_{pr}^{\text{SA}}$. Suppose first that $\varphi(U,V,W)$ is a positive instance of QSAT$_3^{\exists}$. Let $\underline{a} = \langle a_1, a_2, \ldots, a_n \rangle \in \langle \top, \bot \rangle^n$ be an assignment of values to U witnessing this, i.e. the CNF, $\varphi(\underline{a}, V, W)$ is such that every assignment of values to V results in a CNF which is satisfiable. Now consider *any* specific audience, α, for which

$$
\begin{array}{ll}
pos_i \succ_\alpha neg_i & \text{if } a_i = \top \\
neg_i \succ_\alpha pos_i & \text{if } a_i = \bot
\end{array}
$$

Let $U_{\underline{a}}$ be the subset with $u_i \in U_{\underline{a}}$ if $a_i = \top$ and $\neg u_i \in U_{\underline{a}}$ if $a_i = \bot$. From the premise that $\varphi(U,V,W)$ is a positive instance of QSAT$_3^{\exists}$, it is not hard to see that for every consistent subset, $V_{\underline{b}}$ of $\{v_i, \neg v_i \ : \ 1 \le i \le n\}$ (in the sense that exactly one of $\{v_i, \neg v_i\}$ is a member) there is a (consistent) subset, $W_{\underline{c}}$ of $\{w_i, \neg w_i \ 1 \le i \le n\}$ for which

$$S_{\underline{a},\underline{b},\underline{c}} \quad = \quad U_{\underline{a}} \cup V_{\underline{b}} \cup W_{\underline{c}} \cup \{\varphi\} \in \mathcal{E}_{pr}(\mathcal{M}^{(\alpha)})$$

It is certainly the case that this subset is admissible: $S_{\underline{a},\underline{b},\underline{c}}$ is conflict-free in $\mathcal{M}_\varphi$ (therefore trivially so in $\mathcal{M}_\varphi^{(\alpha)}$). Each attack $\neg x$ on some $x \in U_{\underline{a}} \cup V_{\underline{b}} \cup W_{\underline{c}}$ is countered by x itself. The attacks on $W_{\underline{c}}$ from b_3 are countered by $\langle \varphi, b_3 \rangle$. Finally the attacks on φ from C_j are defended by (at least one) argument in $U_{\underline{a}} \cup V_{\underline{b}} \cup W_{\underline{c}}$, specifically the argument matching the literal which satisfies C_j within $\varphi(\underline{a}, \underline{b}, \underline{c})$. Now noting that $S_{\underline{a},\underline{b},\underline{c}}$ is, in fact a *stable* extension, it is immediate that this is a *maximal* admissible set.

It is, furthermore the case that *every* set in $\mathcal{E}_{pr}(\mathcal{M}^{(\alpha)})$ has the form

$$S_{\underline{a},\underline{b},\underline{c}} \ = \ U_{\underline{a}} \cup V_{\underline{b}} \cup W_{\underline{c}} \cup \{\varphi\}$$

where $\varphi(\underline{a}, \underline{b}, \underline{c}) = \top$. For suppose this were not the case, letting, T, be such an example from $\mathcal{E}_{pr}(\mathcal{M}_\varphi^{(\alpha)})$. Noting that for each $u \in U_{\underline{a}}$, as u is unattacked in $\mathcal{M}_\varphi^{(\alpha)}$, it must hold that $u \in T$. It is also the case that for *any consistent* subset, V' of V, $U_{\underline{a}} \cup V'$ is admissible. Now $U_{\underline{a}} \cup V'$ cannot be a maximal admissible set, since, from the premise noting that V' corresponds to an assigment $\underline{b}$, we can identify W', a consistent subset of W for which $U \cup V' \cup W' \cup \{\varphi\}$ is preferred. Thus consider the (non-empty) set $T_{\underline{a},V'} = T \setminus (U_{\underline{a}} \cup V')$. If $\varphi \in T_{\underline{a},V'}$, then the claim that T does not have the required form is contradicted, for any C_j that is unattacked by $U_{\underline{a}} \cup V'$ must be attacked by some w and thence we can build a consistent subset of W to form $W_{\underline{c}}$. If, however, $\varphi \notin T_{\underline{a},V'}$ then, in order for $T_{\underline{a},V'} \neq \emptyset$ to hold we need $T_{\underline{a},V'} \cap \{b_1, b_2, b_3, C_1, \ldots, C_m\} \neq \emptyset$. No argument in $\{b_1, b_2, b_3\}$ is (Dung) admissible, leaving only the possibility that some $C_j \in T_{\underline{a},V'}$. This clause cannot contain any literal corresponding to any $u \in U_{\underline{a}}$ or $v \in V'$. It must, therefore, from the premise that $\varphi(\underline{a}, \underline{b}, W)$ is satisfiable, contain (that is, be attacked by) some argument w or $\neg w$, and hence if $C_j \in T_{\underline{a},V'}$, so T must contain a defence to such an attack. The only available defences, however, are $\neg w$ and b_3: $\neg w \in T$ can only be defended by $\varphi \in T$; $b_3 \in T$, is as we have seen earlier not possible. We deduce that preferred extensions having $U_{\underline{a}} \cup V'$ as a subset must all contain φ. We deduce that if $\varphi(U,V,W)$ is accepted as an instance of QSAT$_3^{\exists}$ then $\langle \mathcal{M}_\varphi, \varphi \rangle$ is accepted as an instance of SBA$^{\text{SA}}$.

Conversely, suppose it is the case that with some value ordering, α, φ is a member of every preferred extension of $\mathcal{M}^{(\alpha)}$. Construct the assignment $\underline{a}$ with $a_i := \top$ if $pos_i \succ_\alpha neg_i$, $a_i := \bot$ if $neg_i \succ_\alpha pos_i$ and consider the CNF, $\varphi(\underline{a}, V, W)$. It suffices to show that $\varphi(\underline{a}, V, W)$ is accepted as an instance of QSAT$_2^{\forall}$. Examining the subset, C' of $\{C_1, \ldots, C_m\}$ with C' given by

$$\{C_1, \ldots, C_m\} \setminus \{ \ C_j \ : \ (u_i \in C_j \wedge a_i = \top) \ or \ (\neg u_i \in C_j \wedge a_i = \bot)\}$$

it is easy to see that C' describes exactly the clauses of $\varphi(\underline{a}, V, W)$. Furthermore the (Dung) standard AF, induced by the arguments

$$C' \cup V \cup W \cup \{\varphi, b_1, b_2, b_3\}$$

is identical to the AF constructed from instances of $\text{QSAT}_2^{\forall}$ defined in [11]. This AF, has the property that $\varphi(\underline{a}, V, W)$ is accepted as an instance of $\text{QSAT}_2^{\forall}$ if and only if $\text{SA}_{pr}(\mathcal{M}_{\varphi}^{(\alpha)}, \varphi)$ from which it follows that if $\langle \mathcal{M}_{\varphi}, \varphi \rangle$ is accepted as an instance of $\text{SBA}_{pr}^{\text{SA}}$ then there is some $\underline{a}$ for which $\varphi(\underline{a}, V, W)$ is accepted as an instance of $\text{QSAT}_2^{\forall}$, i.e. $\varphi(U, V, W)$ is a positive instance of $\text{QSAT}_3^{\exists}$.

The proof of (b) is similar: we again use a correspondence between assignments to a set of propositional variables, U, and specific audiences for a set of values $\{pos_i,\ neg_i\ :\ 1 \leq i \leq n\}$ to establish that $\varphi(U, V)$ – an instance of $\text{QSAT}_2^{\forall}$ – is accepted if and only if an instance $\langle \mathcal{M}_{\varphi}, \varphi \rangle$ is accepted for $\text{OBA}_{pr}^{\text{CA}}$. The UAF, $\mathcal{M}_{\varphi}$, is formed using the standard translation of CNF to AFs from Dimopoulos and Torres [9] as its basis, rather than the construction from [11] used in part (a). We omit the full details for space reasons. Finally, (c) is derived by extending the translation from CNF unsatisfiability used to show OBA is coNP–complete in [7]. □

We conclude our overview of complexity in UAFs by noting some aspects of the case $\sigma = sst$. Recalling from [12] that CA_{sst} is Σ_2^p–complete and SA_{sst} is Π_2^p–complete, it turns out that using the constructions developed therein we obtain:

Lemma 3

a. $\text{SBA}_{sst}^{\text{CA}}$ *is* Σ_2^p*–complete;* $\text{SBA}_{sst}^{\text{SA}}$ *is* Σ_3^p*–complete.*
b. $\text{OBA}_{sst}^{\text{CA}}$ *is* Π_3^p*–complete;* $\text{OBA}_{sst}^{\text{SA}}$ *is* Π_2^p*–complete.*
c. $\text{CA}_{sst}^{\text{OBA}}$ *is* Σ_2^p*–complete;* $\text{SA}_{sst}^{\text{SBA}}$ *is* Π_2^p*–complete.*

Proof: (omitted) □

We note two points of interest from these result: the level of difficulty in $\text{SBA}_{pr}^{\text{SA}}$, $\text{SBA}_{sst}^{\text{SA}}$ and $\text{OBA}_{sst}^{\text{CA}}$. Secondly, the complexity classifications for objective-σ-credulous and objective-σ-sceptical particularly the extreme exhibited when $\sigma = sst$. While it is well-known that in reasoning problems occurring in non-monotonic logics have complexity at the third and higher levels of the polynomial time hierarchy, in the case of the graph-theoretic, *finite* setting of Dung's model (and its developments such as VAFs), the limit of computational difficulty had been Π_2^p and Σ_2^p–completeness. The classifications of $\text{SBA}_{pr}^{\text{SA}}$, $\text{SBA}_{sst}^{\text{SA}}$ and $\text{OBA}_{sst}^{\text{CA}}$ as complete problems at the *third* level of the polynomial hierarchy, indicates that reasoning in uniform argumentation frameworks poses a new level of difficulty.

Regarding the second point of interest. To begin we have the equivalent complexities of $\text{OBA}_{pr}^{\text{CA}}$ and $\text{OBA}_{pr}^{\text{SA}}$: here it is useful first to consider things in rather less technical terms. Thus, $\text{OBA}_{pr}^{\text{CA}}$ seeks to determine whether an argument, x, in a UAF is such that irrespective of the value ordering adopted by individuals, there is *some* defensible case for accepting x. The problem $\text{OBA}_{pr}^{\text{SA}}$, in contrast, is concerned with whether x is such that, irrespective of the value ordering adopted by individuals, the argument x *must* belong to any maximal set of accepted arguments: the classifications demonstrate that whether "some" or "all" maximal sets are relevant, these questions pose identical computational demands (within preferred semantics). In the case of *semi-stable semantics* the distinction between credulous and sceptical variants is even more pronounced: the sceptical version being easier than its credulous form.

5. Contextual Interpretations

We now illustrate the ideas presented in some real contexts. As an informal starting point we can consider the famous quote attributed to Abraham Lincoln: *You can fool some of the people all of the time and all of the people some of the time, but you cannot fool all of the people all of the time.* This would suggest that it is possible to construct an argumentation framework in which an argument was both subjective-sceptical acceptable (so that people with a particular value ordering would always accept it), and objective-credulous acceptable (so that people would see that it was acceptable in some situations whatever their preferences), but not objective-sceptical acceptable.

5.1. Economic Modelling

An important domain in which these distinctions matter is Economics. In an economic model we typically find both *endogenous* variables (those which can be controlled by Government policies, such as the interest rate) and *exogenous* variables (those over which the Government has no control, such as the price of oil). We now represent the arguments concerning exogenous variables as a standard AF, since they are independent of the policy choices, and the arguments about endogenous variables using value-based arguments, since they concern matters that can be chosen according the value preferences of the audiences. Hence,

- Sceptical-objective/objective-sceptical: arguments holding for any policy and any exogenous context. They must be understood, but cannot be influenced by policy. They are hard constraints that policy must adapt to.
- Credulous-subjective/subjective-credulous: these may be acceptable or not according to one's preferences but cannot be controlled.
- Objective-credulous: these will, whatever one's policies, depend on exogenous factors, and so no policy can have an effect.
- Credulous-objective: These relate to issues controlled by exogenous factors, but which are acceptable to all audiences.
- Subjective-sceptical: these are interesting in that it is possible to control their acceptability using a particular preference order. Therefore it is possible to adopt policies that will ensure that they must be accepted.
- Sceptical-subjective: these are also interesting. Whatever the exogenous situation, it is possible to adopt policies that make them acceptable.

Thus the proper concerns of economic policy should be the matters relating subjective-sceptical and sceptical-subjective arguments. Now, however, we can see that there is an important difference between the two. Whereas the subjective-sceptical are matters that can be addressed with a fixed policy, which can be maintained through changing external situations, the sceptical-subjective are issues to which economic policy needs to be responsive: although there is always an answer to changing circumstances, this answer will change over time. The different categories thus indicate where principles can suffice and where flexibility and responsiveness is necessary. Our different categories can therefore be seen as important in economic policy: not only do they identify the proper objects of policy, but they also identify the nature of the polices (principled and unchanging versus responsive and adaptive) that are appropriate.

5.2. Law

Law is another domain in which both directions of fit need to be considered. The typical situation is where a particular case is considered: the facts are determined and the law is applied on the basis of these facts. Normally, we will be concerned with sceptical-objective, sceptical-subjective, credulous-objective, and credulous-subjective. The first two are relatively uninteresting: the facts are clear and on these facts there may or may not be a legal point to argue. Cases before some courts, should always be sceptical-subjective, since points of law are involved: the arguments must be sceptical as all questions of fact are settled and subjective. Often, however, particularly in civil cases where facts are determined not by juries but by the judges, facts may be capable of several interpretations. Here the credulous-objective and credulous-subjective distinction can be related to different jurisprudential attitudes towards the role of the judge. If the argument is credulous-objective, so that there is an interpretation of the facts on which no legal questions arise, adherence to the view that the law should be as clear as possible, and that judges should not exercise more discretion than is necessary, would suggest that that particular interpretation should be adopted. In contrast an upholder of the need for judges to have a wide discretion in order to ensure just outcomes (what detractors of this view call “judge-made law”) would argue that the interpretation should leave some room for values to have influence, and so the facts should be interpreted so as to make the value part of the decision subjective. In the case of credulous-subjective decisions, judges have to to apply a value ordering.

5.3. Practical Reasoning

In practical reasoning we decide what to do, and so the fit is generally from our desires to the world. We are, however, constrained by what it is possible to do in the current situation. So an important part of practical reasoning, before we consider what action should be chosen, is to identify what actions are possible. Much of this concerns arguments with a fit from the world, so the interesting arguments are mostly sceptical-subjective. It should not be overlooked, however, that for all reasonable people there are additional, non-physical, constraints on actions: constraints of morality, or temperament. Thus suppose the problem facing us is that we are short of money: many of us would not regard theft as a possible solution on moral grounds, whereas others would rule theft out only on pragmatic grounds relating to the likelihood of being caught, and a few people might regard theft as the answer. Thus arguments against any plan involving theft are subjective-sceptical accepted: for the first group the argument cannot be defended given their values; the values of the second and third groups in contrast rule it out (or in) on the basis of the facts of the situation.

6. Conclusions

In this paper we introduced a new kind of argumentation framework, *Uniform Argumentation Frameworks*, allowing arguments relating to what is true to be considered along with arguments relating to desires. These are a natural model of many reasoning problems, and provide a way to address domains such as economics and law. The different categorisations offer some interesting insights in these. We have given a complete charaterisation of the complexity issues relating to the decision problems in these frameworks. This characterisation has several interesting aspects, including the classification of $\text{SBA}_{pr}^{\text{SA}}$ as a complete problem at the *third* level of the polynomial hierarchy, a degree of complexity for which very few examples of "natural" graph-theoretic problems are known. As we have seen, however, this category is central to several contexts to which UAFs are applicable.

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