Reasoning Patterns

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Up to now,

- Overview of Machine Learning
- Traditional Machine Learning Algorithms
- Deep learning
- Probabilistic Graphical Models
  - Introduction
  - I-Map, Perfect Map

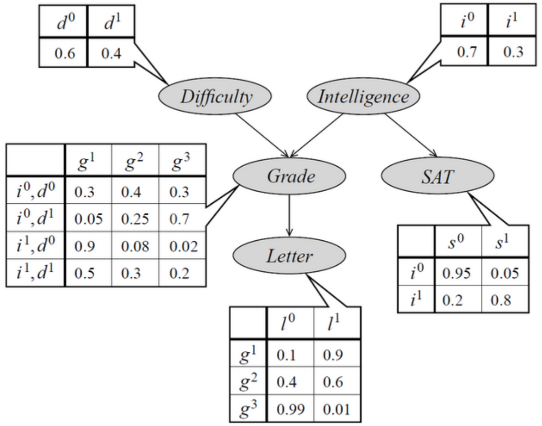
Topics

- Reasoning Patterns
  - Causal Reasoning
  - Evidential Reasoning
  - Intercausal reasoning

Recap: Local Independencies in a BN

- A BN G is a directed acyclic graph whose nodes represent random variables $X_i,..,X_n$.
- Let $Pa(X_i)$ denote parents of X_i in G
- Let $Non\text{-}Desc(X_i)$ denote variables in G that are not descendants of X_i
- Then G encodes the following set of *conditional independence* assumptions denoted Il(G)
  - For each X_i: $(X_i \perp Non\text{-}Desc(X_i) \mid Pa(X_i))$
- Also known as *Local Markov Independencies*

Recap: Local Independencies



- Graph G with CPDs is equivalent to a set of independence assertions

$$P(D,I,G,S,L) = P(D)P(I)P(G \mid D,I)P(S \mid I)P(L \mid G)$$

- Local Conditional Independence Assertions (starting from leaf nodes):

$I(G) = \{(L \perp I,D,S \mid G),$ *L* is conditionally independent of all other nodes given parent G

$(S \perp D,G,L \mid I),$ *S* is conditionally independent of all other nodes given parent *I*

$(G \perp S \mid D,I),$ Even given parents, *G* is NOT independent of descendant *L*

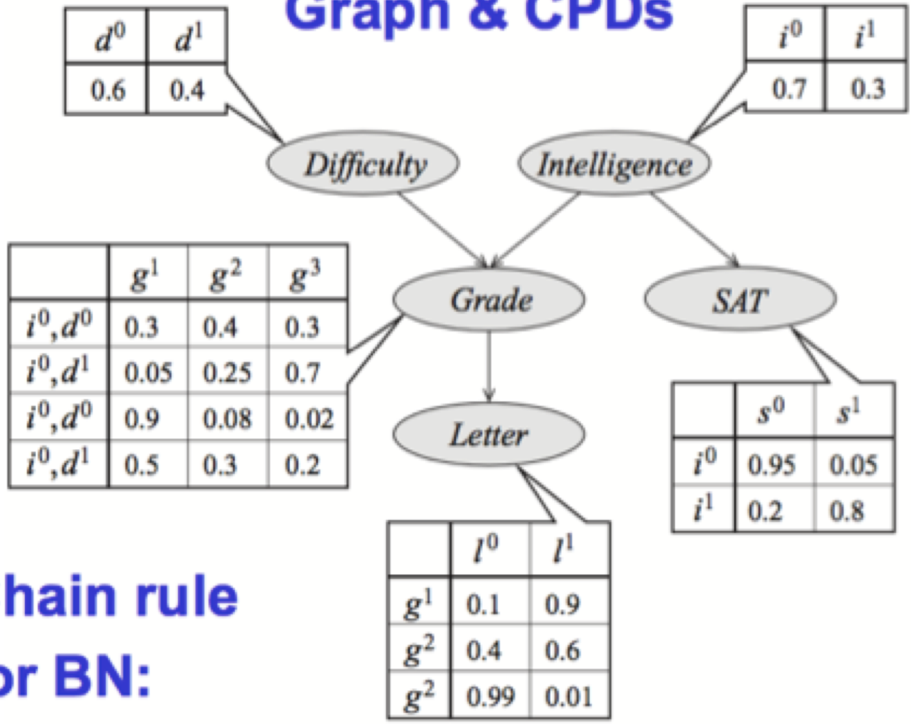
$(\mathrm{I} \perp D \mid \phi),$ Nodes with no parents are marginally independent

$(D \perp I,S \mid \phi)\}$ *D* is independent of non-descendants *I* and *S*

- Parents of a variable shield it from probabilistic influence
  - Once value of parents known, no influence of ancestors
- Information about descendants can change beliefs about a node

Recap: Evaluating a Joint Probability

Graph & CPDs



d^0	d^1
0.6	0.4

i^0	i^1
0.7	0.3

	g^1	g^2	g^3
i^0,d^0	0.3	0.4	0.3
i^0,d^1	0.05	0.25	0.7
i^0,d^0	0.9	0.08	0.02
i^0,d^1	0.5	0.3	0.2

	s^0	s^1
i^0	0.95	0.05
i^1	0.2	0.8

	l^0	l^1
g^1	0.1	0.9
g^2	0.4	0.6
g^2	0.99	0.01

$Val(I)$={i^0=low intelligence, i^1= high intelligence}
$Val(D)$={d^0=easy, d^1=hard}
$Val(G)$={$g^1=A$, $g^2=B$, $g^3=C$}
$Val(S)$={s^0=low, s^1=high)
$Val(L)$={l^0=weak, l^1=strong}

Chain rule for BN:

$$P(D,I,G,S,L) = P(D)P(I)P(G \mid D,I)P(S \mid I)P(L \mid G)$$

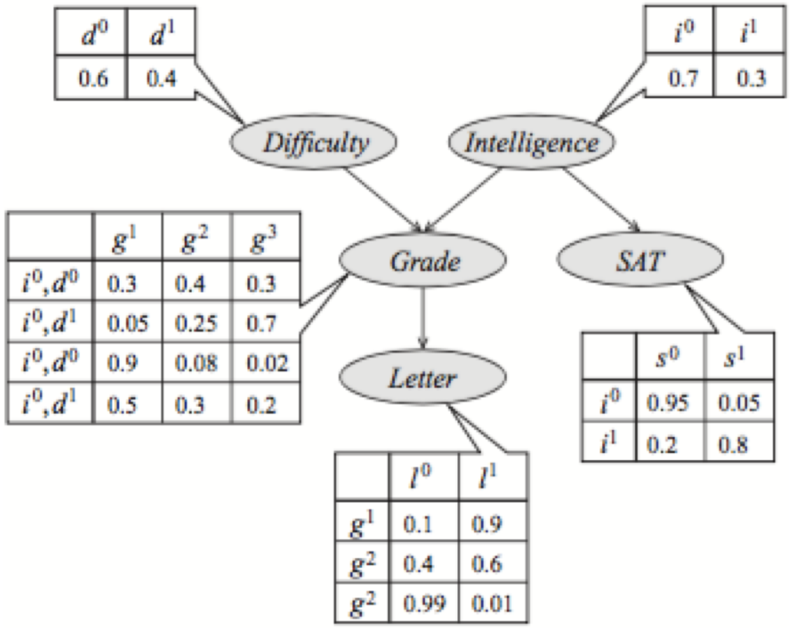
$$P(i^1,d^0,g^2,s^1,l^0) = P(i^1)P(d^0)P(g^2 \mid i^1,d^0)P(s^1 \mid i^1)P(l^0 \mid g^2)$$

$$=0.3\cdot 0.6\cdot 0.08\cdot 0.8\cdot 0.4=0.004608$$

➔ P(high intelligence,
easy course,
grade=B,
high SAT,
weak letter)= very low

Reasoning Patterns

- **Reasoning about a student *George* using the model**



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George

- Causal Reasoning
  - *George* is interested in knowing as to how likely he is to get a strong *Letter* (based on *Intelligence*, *Difficulty*)?
- Evidential Reasoning
  - *Recruiter* is interested in knowing whether *George* is *Intelligent* (based on *Letter*, *SAT*)

Recruiter

Causal Reasoning

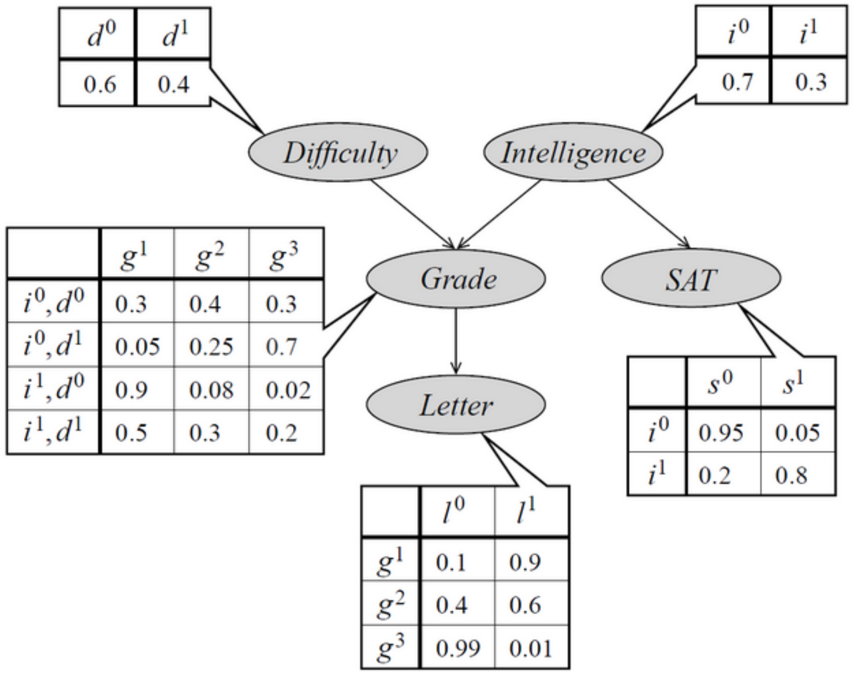
Causal Reasoning

- How likely *George* will get a strong *Letter* (No evidence)?

$$P(l^1) = \sum_{D,I,G,S} P(D,I,G,S,L=l^1) = \sum_{D,I,G,S} P(D)P(I)P(G|D,I)P(S|I)P(l^1|G)$$

- $P(l^1)$=0.502
- Obtained by summing-out other

variables in joint distribution



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Causal Reasoning

- Knowing *George* is not so *Intelligent* (i^0)

$$P(l^1|i^0) = \frac{P(l^1, i^0)}{P(i^0)} = \frac{\sum_{D,I,G} P(D)P(i^0)P(G|D,i^0)P(S|i^0)P(l^1|G)}{\sum_{D,G,S,L} P(D)P(i^0)P(G|D,i^0)P(S|i^0)P(L|G)}$$

- $P(l^1|i^0)$=0.389

$P(l^1)=0.502$

$P(l^1 \mid i^0)=0.389$

After knowing that the student is not as intelligent, we understand that the probability of getting a strong recommendation letter is lower.

So, when the employer received a strong recommendation letter, what does this mean?

Causal Reasoning

- Knowing COMP219 is not *Difficult* (d^0)
- $P(l^1 | i^0, d^0)$=0.513 (Exercise!)

- Observe how probabilities change as more evidence is obtained
- ***Causal Reasoning***:
  Predicting downstream effects of factors such as *Intelligence*

$P(l^1)=0.502$

$P(l^1 \mid i^0)=0.389$

$P(l^1 \mid i^0, d^0)=0.513$

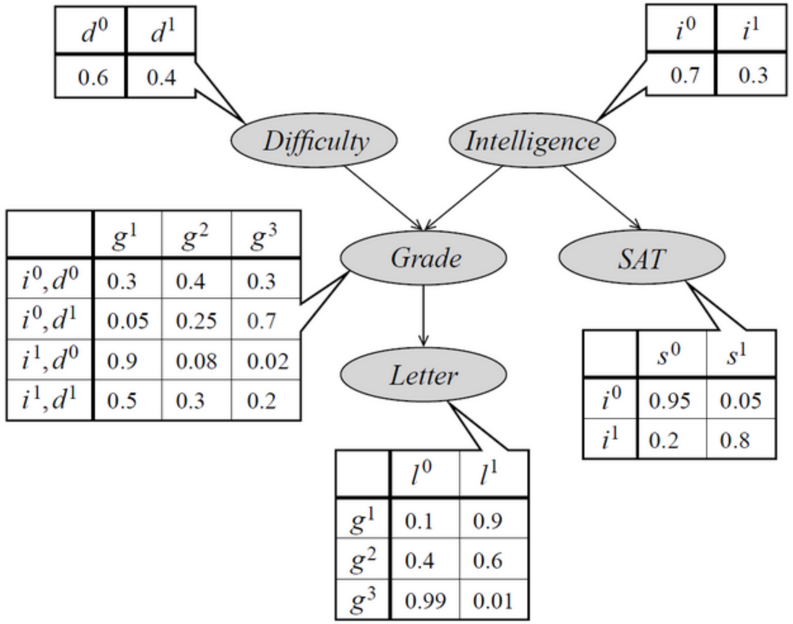
After knowing that the student is not as intelligent, we understand that the probability of getting a strong recommendation letter is lower.

After further knowing that the difficulty is low, the probability of getting a strong letter is higher.

So, when the employer received a strong recommendation letter, what does this mean?

Evidential Reasoning

Evidential Reasoning



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- Recruiter wants to hire *Intelligent* student
- A priori *George* is 30% likely to be *Intelligent*

  $P(i^1)=0.3$
- Finds that *George* received *Grade C* (g^3) in *COMP219*

  $P(i^1|g^3)=0.079$
- Similarly probability of *Difficult* goes up from 0.4 to

  $P(d^1|g^3)=0.629$
- If recruiter has lost *Grade* but has *Letter*

  $P(i^1|l^0)=0.14$

$$
\begin{aligned}
P(i^1|g^3) &= \frac{P(i^1, g^3)}{P(g^3)} \\
&= \frac{\sum_{D,S,L} P(D)P(i^1)P(g^3|D,i^1)P(S|i^1)P(L|g^3)}{\sum_{D,I,S,L} P(D)P(I)P(g^3|D,I)P(S|I)P(L|g^3)}
\end{aligned}
$$

low grade drastically decreases the probability of high intelligence

low grade justifies the difficulty

$$P(i^1)=0.3 \longrightarrow P(i^1 | g^3)=0.079 \longrightarrow P(d^1 | g^3)=0.629$$

$$P(i^1)=0.3 \longrightarrow P(i^1 | l^0)=0.14$$

A weak letter drastically decreases the probability of high intelligence

Evidential Reasoning

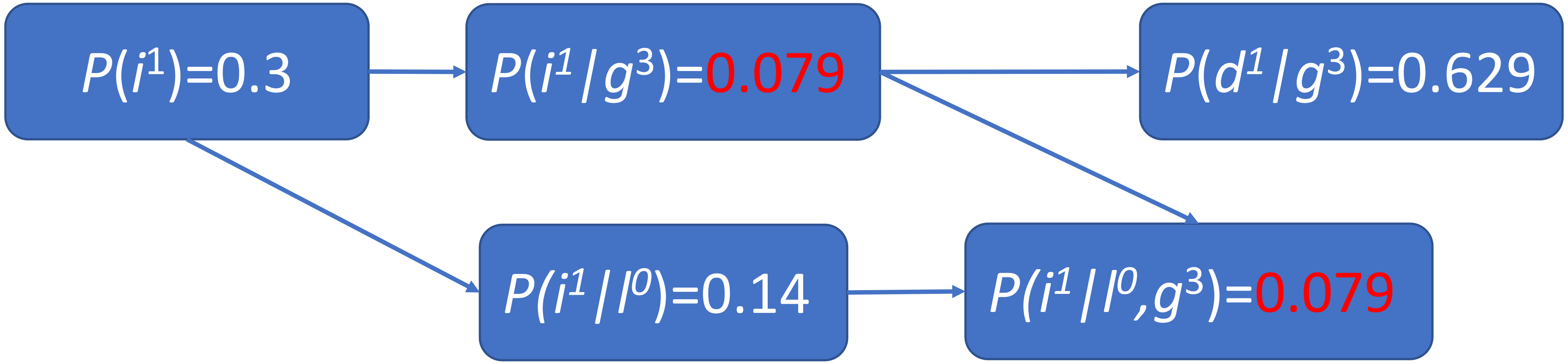
- Recruiter has both *Grade* and *Letter*

  $$P(i^1 | l^0, g^3) = 0.079$$

  - Same as if he had only *Grade*
  - *Letter* is immaterial

- Reasoning from effects to causes is called evidential reasoning low grade drastically decreases the probability of high intelligence

low grade justifies the difficulty



A weak letter drastically decreases the probability of high intelligence

After knowing low grade, a weak letter won’t make the probability of high intelligence lower.

Intercausal reasoning

Intercausal reasoning

- Recruiter has Grade (*Letter* does not matter for *Intelligence*)

  $P(i^1|g^3)=P(i^1|l^0,g^3)=0.079$

- Recruiter receives high *Score* (leads to dramatic increase)

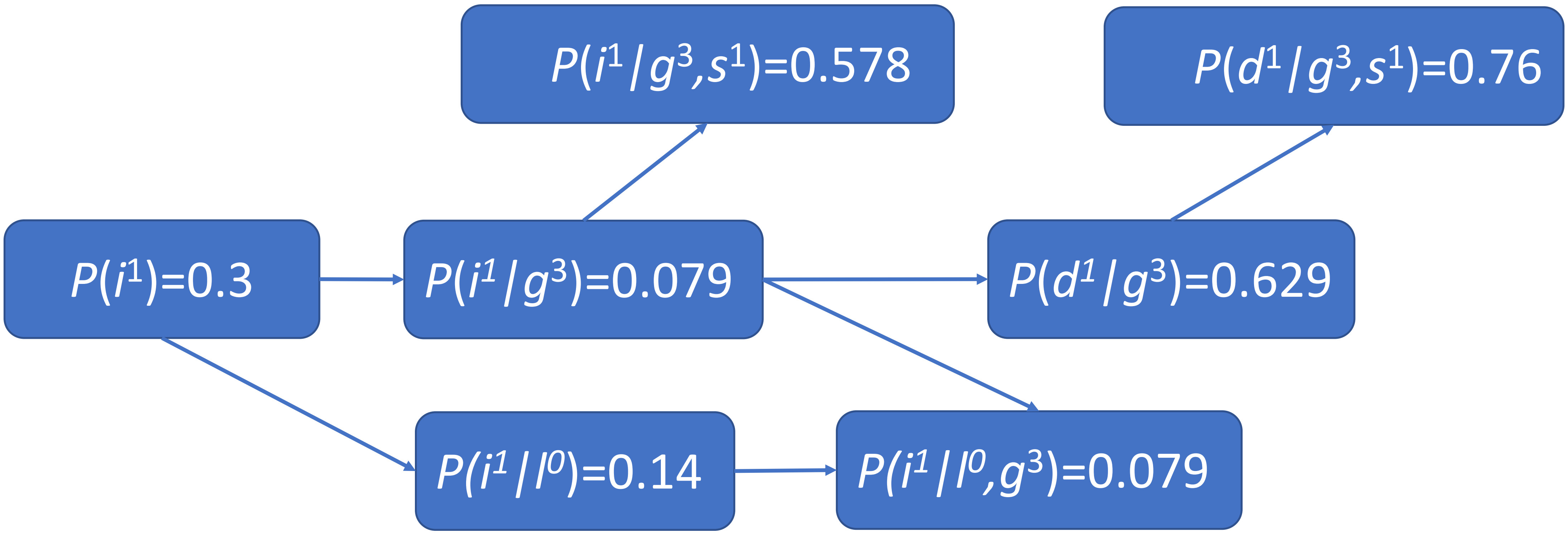
  $P(i^1|g^3,s^1)=0.578$

- Intuition:
  - High Score outweighs poor grade since low intelligence rarely gets good Scores
  - Smart students more likely to get Cs in hard classes

- At the meantime, Probability of class is difficult also goes up from
  - $P(d^1|g^3)=0.629$ to
  - $P(d^1|g^3,s^1)=0.76$

High Score outweighs poor grade since low intelligence rarely gets good Scores

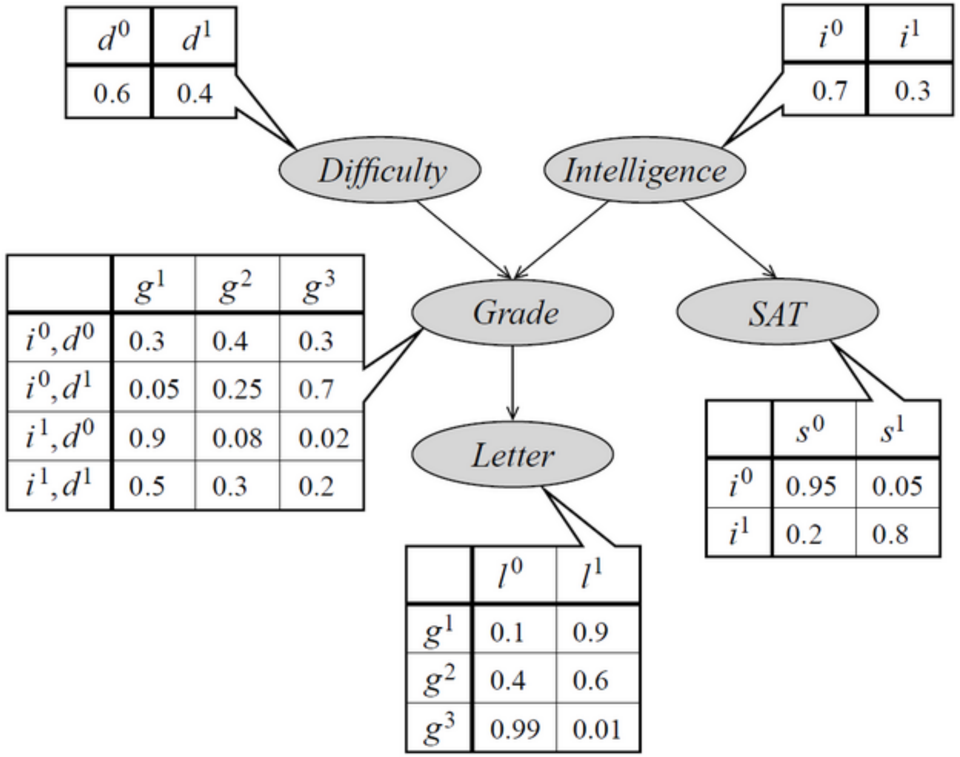
Probability of class is difficult also goes up



Intercausal reasoning

- The previous example:
  - Information about Score gave us information about Intelligence which with Grade told us about difficulty of course
  - One causal factor for Grade (Intelligence) give us information about another (Difficulty)

Explaining Away



- Given *Grade=C, Letter=weak*

  $$P(i^1|g^3)=0.079$$

- If we observe *Difficulty=high*

  $$P(i^1|g^3,d^1)=0.11$$

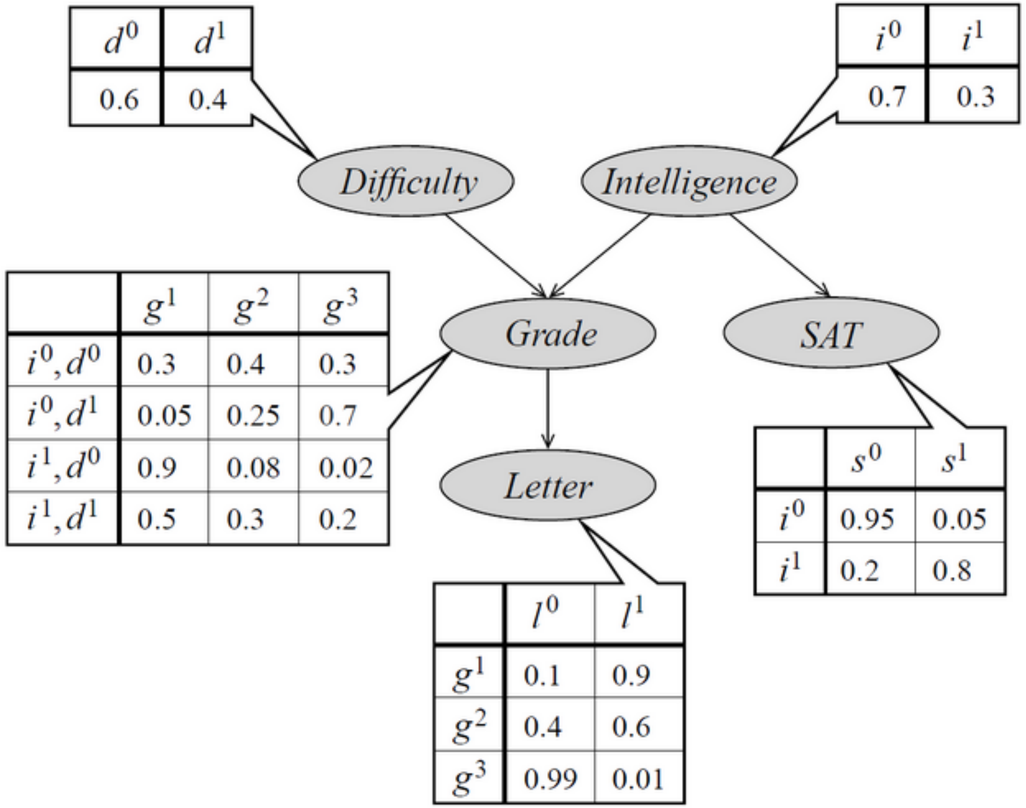
- We have provided partial explanation for George's grade in COMP219

0.11 < 0.3 :   partial explanation
for George's grade

$$P(i^1 \mid g^3, d^1) = 0.11$$

$$P(i^1) = 0.3 \rightarrow P(i^1 \mid g^3) = 0.079 \rightarrow P(i^1 \mid g^3, d^1)$$

Explaining Away



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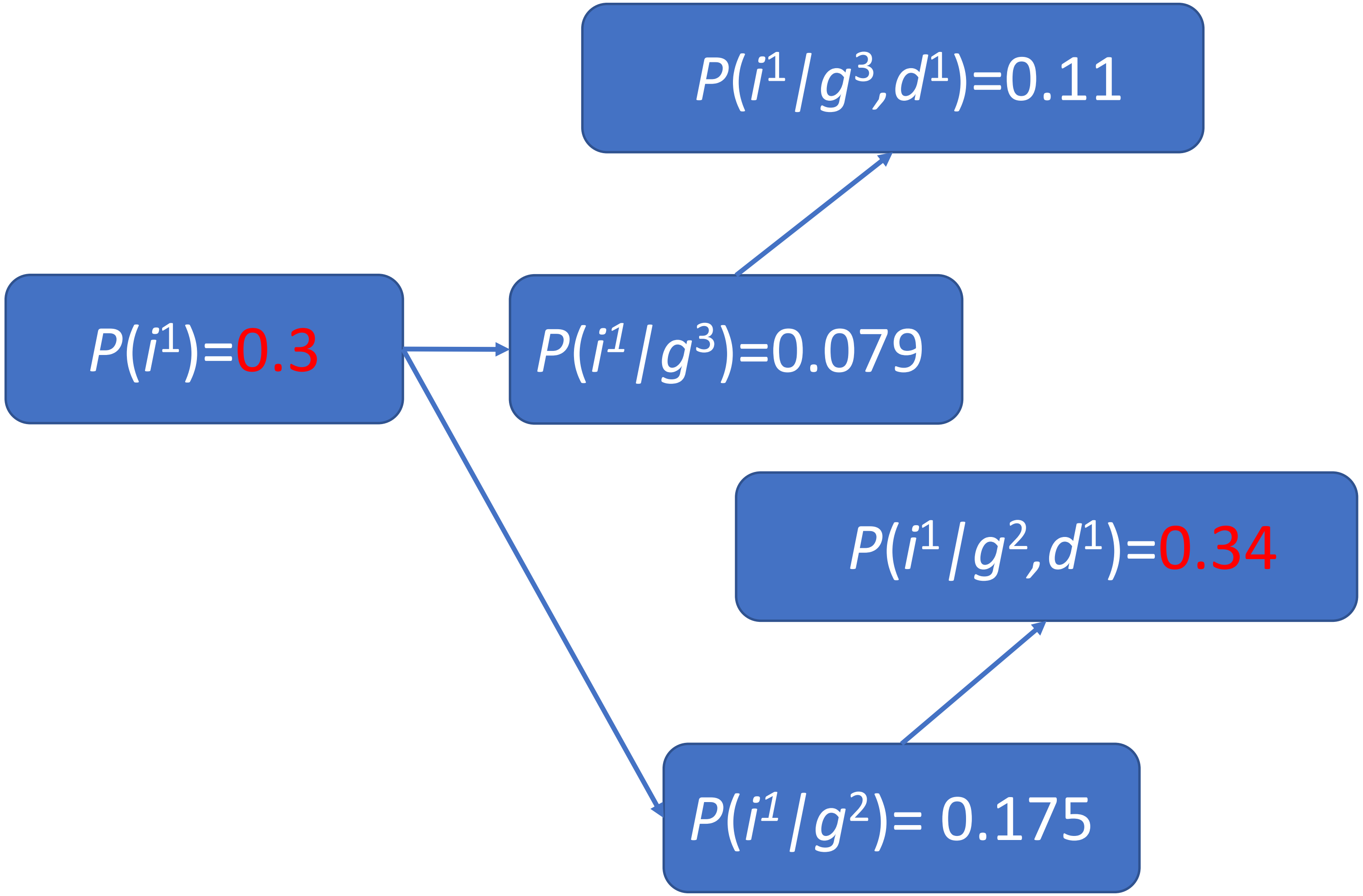
- If George gets a *B* in COMP219

$$P(i^1 | g^2)=0.175$$

- If we observe COMP219 is hard

$$P(i^1 | g^2,d^1)=0.34$$

- We have explained away the poor grade via the difficulty of the class partial explanation for George's grade



0.34 > 0.3:
explained away the poor grade via the difficulty of the class

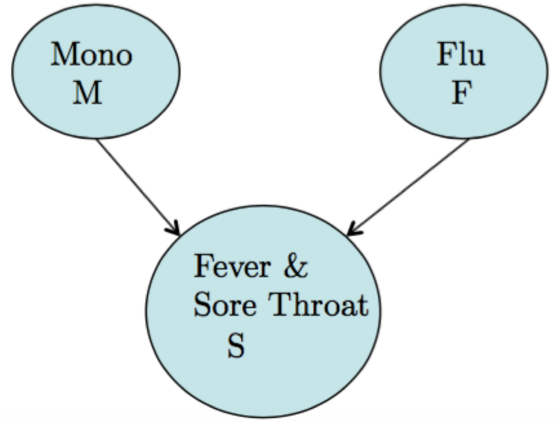
Explaining Away

- Explaining away is one type of intercausal reasoning
- Different causes of the same effect can interact
- All determined by probability calculation rather than heuristics

Common in Human Reasoning

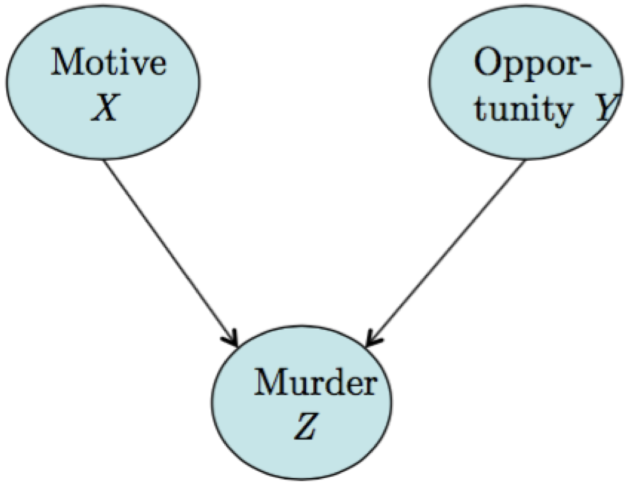
- Binary Variables
- Fever & Sore Throat can be caused by mono and flu
- When flu is diagnosed probability of mono is reduced (although mono could still be present)
- It provides an alternative explanation of symptoms

$$P(m^1|s^1)>P(m^1|s^1,f^1)$$



Another Type of Intercausal Reasoning

- Binary Variables
  - Murder (leaf node)
  - Motive and Opportunity are causal nodes
- Binary Variables *X,Y,Z*
- *X* and *Y* both increase the probability of Murder
  - $P(z^1|x^1)>P(z^1)$
  - $P(z^1|y^1)>P(z^1)$
- Each of *X* and *Y* increase probability of the other
  - $P(x^1|z^1)<P(x^1|y^1,z^1)$
  - $P(y^1|z^1)<P(y^1|x^1,z^1)$



Can go in any direction
Different from Explaining Away